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Bengt Norén:

Nailed Joints — Their Strength and Rigidity
under Short-Term and Long-Term Loading



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Nailed Joints — Their Strength and Rigidity under Short-Term and Long-Term Loading

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Several theories have been published regarding the performance of nailed joints, covering both linear and non-linear slip in the joints and their strength. In earlier publications the author has examined and in part extended them, e. g. as regards the influence of the duration of loading. However, fundamental data for the practical application of the theories, particularly as regards the compliance and strength of the timber under nails, have been lacking. The purpose of the tests described has been to determine such bed moduli and their dependence on such variables as type and diameter of the nails, density, moisture content and fibre direction of the timber, and the duration of loading.

The majority of the short-term tests have been adapted to analysis of variance. At the long-term tests the variables have been limited to the diameter of the nails and the angle between force and fibre direction. In addition to displacement tests, they included some tests with three-member joints. The phenomenological description of creep and recovery has received particular attention. The creep function of the well-known mechanical model represented by Kelvin bodies in series has been modified with regard to the irreversible creep.

The results of the short-term tests show simple relations between the independent variables as regards the characteristic moduli of displacement and strength as well as more complicated interactions. However, it has been possible to give standard values for the practical calculation of the slip in and strength of nailed joints which are linearly dependent on the density of the timber and in some cases on the diameter of the nails. The values have been used for the calculation of e. g. slip moduli and the results compared with test results obtained by the author and others.

The long-term tests clearly showed the difference between initial creep and creep at repeated loading. The creep was distinctly non-linear as regards the load as well as the instantaneous displacement. The reversibility of the creep was considerably greater perpendicular to the fibre direction than parallel. However, the relative creep in joints with slender nails was found to be less dependent on the load than the corresponding displacement creep, which may be ascribed to the retarding effect of the bending of the nails.

Among the practical results of the investigation it may be mentioned that time-dependent slip moduli are given for nailed joints and that their application at the calculation of the deformation and stresses in nailed beams is demonstrated.

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PREFACE

In a paper dealing with the theory of nailed joints presented by the author at the First International Conference on Timber Engineering in Southampton in 1961 (Norén, 1962), the strength and rigidity of the joints, including creep, were attributed to particular strength properties of the wood and the nails.

However, the author had already several years ago carried out tests in which the reaction of wood exposed to stress from laterally loaded nails was studied. The results from these tests are presented in detail in the first part of this report.

The displacement tests were continued by long-term loading to determine the creep. These tests are dealt with in the second part, which on the whole coincides with a paper presented by the author at the International Symposium on Joints in Timber Structures, London 1965.

In the third part the author discusses slip and creep in nailed joints designed, as is usually the case in practice, with a high ratio of timber thickness to nail diameter. Results from long-term tests are presented.

The fourth part, finally, is devoted to the slip modulus of nailed joints and its practical application.

The Author's tests were carried out at the Swedish Forest Products Research Laboratory, part of the costs being paid by the National Swedish Council for Building Research. The author is greatly indebted to a number of persons for their assistance. Among other we want to thank B. Thunell, Director of the Wood Technology, Department of the Laboratory, and E. Saarman for his valuable help in analysing the test results.

Stockholm, June 1968

Bengt Norén

LIST OF SYMBOLS

Cross section quantities and other dimensions

a	Spacing of nails
b, c	Thickness of layers in composite beam
d	Diameter (of nail or bolt)
h	Width of member (in composite beam)
l, m, s	Thickness of timber (in joint)
l	Also span of beam
l_e	Effective thickness of timber (or effective length of nail)
r	Spacing of members (in composite beam) (= distance between centroids). See also "Other symbols"
A	Cross sectional area (of wood member)
A_1, A_2	Levels of d values, see table 1
A_r	Fictitious area defined by (69)
S	First area moment. See also "Forces"
S_1, S_2, S_3	Nail sections: round, square (grooved) and square diagonally to the force, see table 1
I	Second area moment (moment of inertia)

Loads, Forces and Moments

P	Load on nailed joint
q	Load per unit length of nail (bolt) or beam
R	Transverse shear force. See also "Other symbols"
S	Shear force on nail in the joint
T	Resulting shear force in the joint
N	Resulting force perpendicular to a cross section (of nail or timber)
M	Bending moment

Stresses

σ	Compressive stress in wood under nail (bolt).
	Specified values are:
σ_p	Stress at proportional limit

$\sigma_{2(4)}$	Stress at 2 mm (4 mm) displacement of nail
$\sigma_{0.02d}$	Stress giving a displacement of which $0.02d$ is "permanent"
σ_F	Stress giving displacement flow
σ	Compressive (or tensile) stress in wood parallel to the grain
σ_B	Compressive strength of wood parallel to the grain
σ_b	Bending strength of nail
τ	Shear stress. See also "Other symbols"

Displacement, deformation and strain

y	Lateral displacement of nail
y	Also deflection of beam
y_{00}	Instantaneous displacement at the joint ($t = 0, x = 0$)
g	Slip in the joint
ψ	Specific slip. See also "Other symbols"
a_i	Creep "amplitude" defined by (27)
p	Coefficient of stress distribution (in composite beam)
	$\frac{dp}{dx}z =$ strain in cross section at z -level
ε	Strain

Coefficients of elasticity and plasticity

E	Young's modulus
G	Rigidity modulus
K	Displacement modulus
k	Slip modulus
η	Coefficient of viscosity
α	Ratio of elastic displacement (slip) to total displacement (factor of recovery)

Other symbols

φ	Ratio of strength of wood under nail to strength of prism
φ	Also magnification factor for stress in composite beam
Φ	Stress function
ψ	Magnification factor for deflection of composite beam
ψ	Also specific slip
Ψ	Time function
β^2	Ratio of sum of moment of inertia of individual members to moment of inertia of total composite cross section

κ	Factor defined by (71a)
ω	Factor defined by (77b)
ν	Factor defined by (82)
C_ψ	$\frac{1}{\beta^2} - 1$
C_i	Factor defined by (84)
r_{0u}	Density of wood (weight of absolutely dry wood, volume at moisture content u)
u	Moisture content (weight of water/weight of dry wood)
u	Also stiffness factor defined by $u = \sqrt[4]{\frac{K}{4EI}}$
$R_1 R_2 R_3 R_4$	Levels of density, see table 1
$F_1 F_2 F_3$	Levels of moisture content, see table 1
um, ul	Slenderness of nail
n	Number of members
m	Number of joints
t	Time
T	Time ($1/2T$ = frequency)

Indices

$\parallel$	Parallel to the fibres
$\perp$	Perpendicular to the fibres
G	Spruce
F	Pine
H	Hooke
N	Newton
0	Instantaneous ($t = 0$) or "at the joint", cf. y_{00}
s or sec	Secant (modulus)
e	Effective, equivalent
r	Recovery
R	Reference
M	Mean
c	Creep
I	Initial
w	Working load (stress)

DEFORMATION AND STRENGTH OF TIMBER UNDER COMPRESSION FROM NAILS

Introduction

In a previous report (Norén, 1962) the author dealt with the theory of nailed joints. The strength of nailed joints depends on the compressive strength of the timber under the nails, which is akin to and can generally be related to the strength of the timber parallel to the grain defined by standard tests.

The compression is defined by

$$\sigma = \frac{q}{d} \quad (1)$$

where q is the force per unit length of the nail and d the diameter of the nail. Similarly, the slip in the joint within the area where the deformation can be regarded as proportional to the force, depends on the "displacement modulus" defined by

$$K = \sigma \frac{d}{y} \quad (2)$$

where y is the lateral displacement of the nail in the timber under load. (The displacement is assumed to be perpendicular to the length of the nail. Forces and displacements in the direction of the length of the nail are not considered here.)

The coefficients (or corresponding factors) here defined have long been regarded as an important basis for the determination of the technical properties of timber joints. The correlation between the compressive strength of the timber under the nails and the strength of nailed joints was defined more than 25 years ago (Johansen, 1941)¹ and in the elasticity theory the expressions for the deformation of nailed joints had been derived long before that. However,

the way in which these coefficients vary with certain important factors remained unknown. Moreover, the proportional limit in the load-slip curve of timber joints, particularly nailed joints, is often very low. At the same time, the ultimate load is associated with considerable displacements in the joint. The strength under the nail and the displacement modulus alone will therefore give too limited an idea of the behaviour of nailed timber joints.

Of the coefficients for nailed joints, the one most often found in the literature on the subject is the compressive strength σ_B . Sometimes the compressive stress at a certain lateral displacement is given, σ_y , and occasionally, also the strength is defined by such a value. Data on the actual displacement modulus, K , are only rarely found. Quite apart from differences in woods, density, moisture content, type of nail, etc. it is often very difficult to compare values from different sources. Sometimes, for example, the ultimate stress under compression is insufficiently defined. (The strength is also given in relation to the strength determined on prisms or cubes. Also these control tests are not always quite comparable.) The most reliable coefficients are those determined by a direct method, so-called displacement tests. Coefficients indirectly determined from tests on timber joints of such dimensions that the bending of the nail influences the distribution of the stress are often based on an assumed distribution of stress along the nail which differs considerably from the actual distribution. Their values will therefore depend on the design of the joint—timber thickness, number of members, number of nails, etc.—in a manner that is not consistent with true coefficients relating to material.

Among the earlier investigations of the compressive strength under nails are those carried out by Stoy, 1935. Although he worked with relatively high slenderness ratios and a greatly simplified stress distribution and although he partly used exposed nails, i. e. not driven into the timber, certain results may be used as a basis for a comparison with the

¹ Preuss, 1922, see also de Rover, 1964, defined the breaking load for double-shear joints as $P = Cd^2 \sqrt{\sigma_b} \sigma$, where σ_b is the bending strength of the fastener and σ the compressive strength under the nail. Owing to the particular longitudinal distribution of σ assumed by Preuss, he obtained a somewhat lower C value ($C = 0.775$) than the values corresponding to the flow theory later generally used (Möller, 1951) which for round sections gives $C = 0.883$.

results presented in the present report. The strength perpendicular to the grain was equal to or greater than the strength parallel to the grain. Round nails resulted in lower strength than square nails. For the latter nails, $\varnothing$ 2.8—4.6 mm, the ratio of the strength under the nails to the prism strength may be calculated to be $\varphi = 0.8$ for dry timber and $\varphi = 1.12$ for green timber (Norrefeldt, 1945). According to Stoy, the value of φ diminishes with increasing slenderness (ratio of timber thickness to nail diameter) and with decreasing moisture content. However, these results may be assumed to be unduly influenced by the bending of the nails and do not permit of any reliable conclusions as to the extent to which the compressive strength or other relevant coefficients may be dependent on the diameter of the nails.

Möller, 1951, has, by testing grooved¹ wire nails with a diameter of 2.8—8.0 mm driven into pine and loaded parallel to the grain, obtained φ -values which show a slight tendency to rise with decreasing nail diameter. For displacement tests in the strict sense, the mean values given are $\varphi = 0.75$ for grooved wire nails, and $\varphi = 0.74$ for nailed joints in single shear. The mean value obtained for round wire nails was $\varphi = 0.49$. The values are comparatively low, which is explained by the fact that the strength values obtained have been multiplied by the factor 0.85 in order, at least to some extent, to compensate for the short duration of loading. The value 0.75, according to fig. 19, coincides with the relation between the compressive stress at 2 mm displacement of 5 mm nails and the strength of prism found by the author. Möller's values for the compressive strength under round nails—65 % of the values for grooved nails—are particularly low. The results in figs. 11 a, b show a smaller dependence on the section of the nail, in accordance with the results previously published by the author (Norrén, 1955).

Simultaneously with the tests carried out by the author (stage 1), Siimes, Johansson & Niskanen, (1954) made displacement tests in order to determine the compressive strength under nails. These tests were made on pine and mainly with plain square wire nails and test specimens (double-shear joints with side members of steel) similar to those used by the author. At an

initial stage an attempt was made to imitate the nail by using a T- or U-section piece of steel shaped to correspond to a nail of round and square section, respectively. The T-shaped piece was driven into a groove cut out in the specimen. The method, which has previously been used, e. g. by Johansen, 1941, (to determine the compressive strength under dowels and bolts) and (without groove) by Meyer, 1957 and Mlynek, 1954, is less suitable for determining the compressive strength under nails. The strength for round sections parallel to the grain (density $r_{0.04} = 0.45$ g/cm³ moisture content $u = 0.15$) obtained by the T-section tests was 77 % of the strength for square sections. This is about the same result as that obtained by the author, figs. 11 a, b, moisture content level F_1 . The Finnish results according to (Siimes *et al.*, 1954) are given in fig. 1. The strength values have been converted to relative values by dividing them by the value corresponding to the dry density 0.445 g/cm³ (the density has been recalculated from the density of green timber to dry density). Apart from a tendency towards increased dependence on the density as the density increases, the dependence on the density of the compressive strength according to the Finnish results on the whole agrees with the Swedish test results (broken line in the figure). No significant interaction between the density and the nail diameter or the moisture content, within the range 8—22 %, could be detected. Fig. 2 shows the diameter dependence of the strength parallel to the grain according to Siimes *et al.*, 1954. With the exception of a series with dry timber and high density (dot-and-dash curve), the diameter dependence increases with the diameter of the nail. The linear diameter dependence reproduced from the result obtained by the author, (fig. 15), according to the Finnish results, exaggerates the strength at diameters over 5 mm.

The Finnish tests also show that the angle between the longitudinal direction of the nail and the annual rings had a certain influence on the strength parallel to the grain. Nails driven tangentially to the annual rings gave a somewhat lower strength than radially driven nails. Nails driven at an angle of about 45° to the annual rings gave intermediate values. The dependence of the strength on the angle between the direction of the force and the grain was also investigated. The lowest strength—in this case defined as the compressive stress at 1.5 mm displacement—was obtained parallel to the grain and the highest perpendicular to the grain. Compression at an angle of 45° to the grain gave intermediate values. For

¹ Longitudinally grooved = fluted.

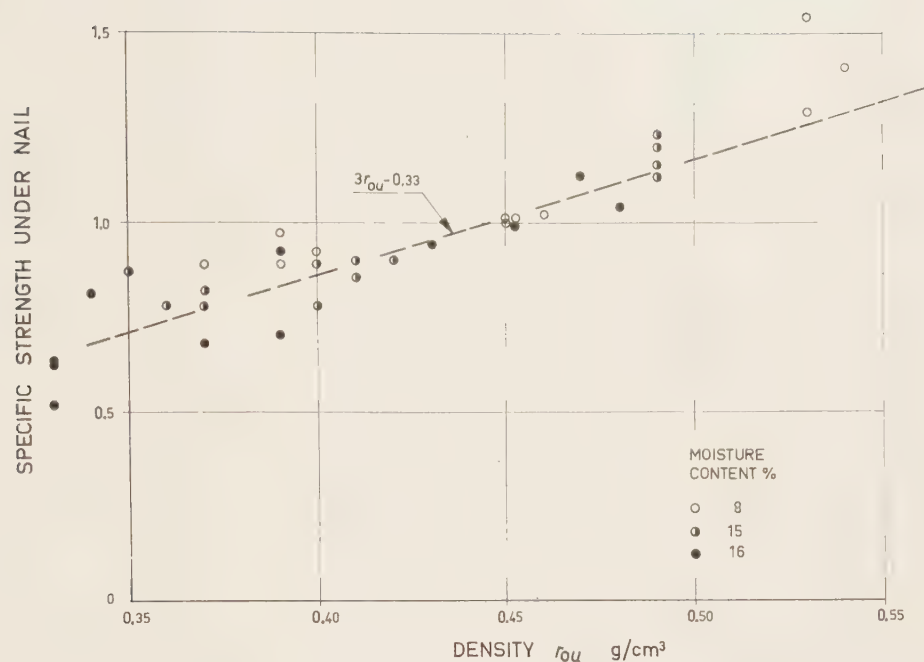


FIG. 1. Relation between the density and the compressive strength under square nails in pine. Results from Finnish tests (Siimes *et al.*, 1954) (circles) agree with the mean result obtained by the author (broken line).

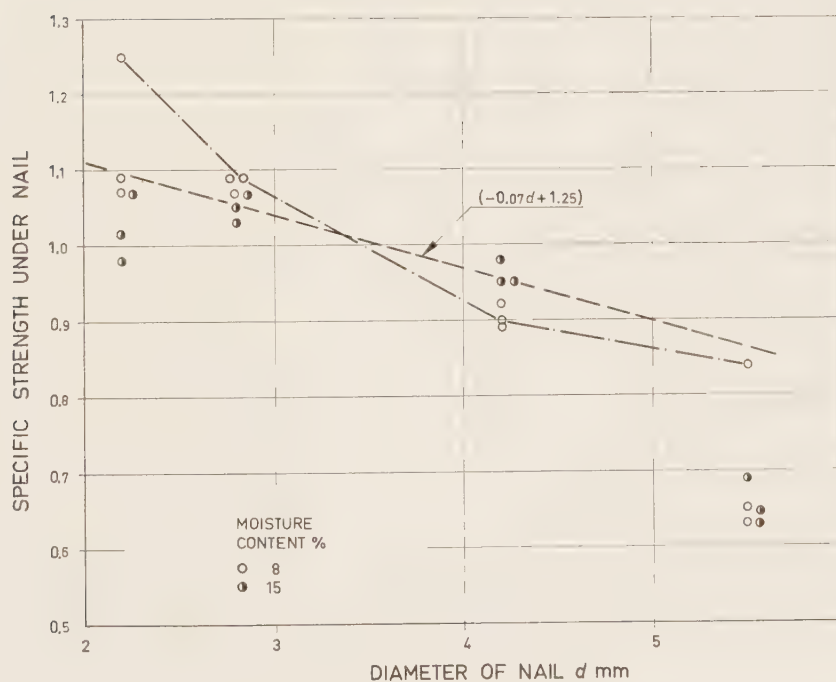


FIG. 2. Diameter dependence of the strength under nails at loading parallel to the grain. Results from Finnish tests (Siimes *et al.*, 1954). Cf. fig. 1. The broken straight line shows the corresponding diameter dependence obtained by the author.

dry timber, the Finnish results are entirely comparable with the results obtained by the author (fig. 12). For green timber, the Finnish tests showed a considerably smaller dependence on the direction than the Swedish results. This may in part be due to the fact that the compressive stresses in the Swedish tests were compared at a slightly greater displacement (2 mm).

Comprehensiveness of the tests and procedure

The purpose of the short-duration tests described in this section was to supplement available data on the strength and the displacement modulus of pine and spruce under ordinary wire nails. The proportional limit and a few other points on the curve showing the relation between the compression and the displacement were also studied.

The tests were made by two stages, the first with several variables and the second with a more limited number of variables. The former experiments were carried out as factor tests for analysis of variance. In the final analysis the test results were divided into four principal groups:

- F $\parallel$ Pine parallel to the grain
- F $\perp$ Pine perpendicular to the grain
- G $\parallel$ Spruce parallel to the grain
- G $\perp$ Spruce perpendicular to the grain.

Within these groups the remaining factors were varied as shown in table 1. The levels give 81 combinations in each group. Each combination was tested twice and the number of tests in this part of the investigation was thus 648. In addition, round nails and bolts (dowels) in prebored holes were also tested. The variables and levels are given in table 2. Also in this case, each combination was tested twice. However, the combinations with light and heavy timber were not complete. The number of tests amounted to 364. Altogether, some 1000 displacement tests were thus carried out at this stage.

The densities given in tables 1 and 2 are mean values. The maximum deviation within each level is ± 0.03 g/cm³. The value $u = 30\%$ indicates the lowest moisture content. The temperature was 20°C.

The timber dimensions and the diameter of the fasteners are given in table 3. As regards the construction of the specimens, it may be mentioned that the

TABLE 1. Variables at displacement tests on nailed joints, pine and spruce. Force $\parallel$ and $\perp$ to the grain.

Variables	Levels	Comments
Dry density (g/cm ³)	R ₁ 0.33	Only spruce
	R ₂ 0.39	
	R ₃ 0.45	
	R ₄ 0.51	
Moisture content (%)	F ₁ Nailed and tested at 12	Only pine
	F ₂ Nailed and tested at 30	
	F ₃ Nailed at 30, tested at 12	
Section of nail	S ₁ Round (SMS 1384) ^a	
	S ₂ Square, grooved (SMS 1382) ^a	
	S ₃ Do. "diagonally to the grain"	
Diameter of nails (mm)	A ₁ 2.3	The values deviate somewhat, table 3
	A ₂ 3.2	
	A ₃ 5.0	

^a Swedish Standard.

TABLE 2. Variables at displacement tests on pine and spruce joints with nails and bolts in prebored holes. Force $\parallel$ to the grain.

Variables	Levels	Comments
Dry density	R ₁ 0.33	R ₁ , R ₄ and R ₅ are not fully represented
	R ₂ 0.39	
	R ₃ 0.45	
	R ₄ 0.51	
	R ₅ 0.57	
Moisture content (%)	F ₁ 12	} At nailing and testing
	F ₂ 30	
Diameter (mm)	A ₁ 2.1	Diameter of bits, mm ^a
	A ₂ 3.5	
	A ₃ 5.1	
	A ₄ 10	
	A ₅ 17	
	A ₆ 25	

^a Special bits were used to give holes closely corresponding to the diameter of the bits.

TABLE 3. Dimensions of fasteners and timber. Symbols according to fig. 3.

Nails, bolts		Timber		
Section	d mm	t mm	b mm	b mm
S ₁ (round)	2.1	8	50	100
	3.5	13	50	100
	5.0	20	70	140
	10	37	70	140
	17	60	100	175
	25	90	100	175
S ₂ , S ₃ (grooved)	2.3	8	50	100
	3.2	13	50	100
	5.0	20	70	140

central members for the bolts, in order to make the timber as uniform as possible, were glued together (laminated) of 20 mm thick laminae. The same timber dimensions were used for compression tests parallel to the grain and perpendicular to the grain. Parallel to the grain, all pieces of timber were first sawn up in lengths twice as long as the specimens used. The pieces were then halved and one half used for the displacement test and the other for determining the density, the moisture content and the strength parallel to the grain. The compressive strength was determined on prisms measuring in mm $t \times 20 \times 3 t$ —maximum $20 \times 20 \times 60$. All prisms were tested at a moisture content of 12 % and a constant compression rate of 1 mm/min.

The test specimens can be described as nailed double-shear joints with side members of steel and the central member of wood. The thickness of the central member was chosen with a view to minimum bending of the nail in relation to the displacement. For the same reason, the steel members were com-

paratively thick and provided with bushings adjusted to the section of the fastener, which provided for firm clamping of the ends of the fastener. The test specimens were made in such a way that the nails were driven through the timber. A specially designed jig was used to guide the nails in the direction intended perpendicular to the flat side of the timber and to orientate the square nails correctly in relation to the grain (S_2 and S_3).

The compressive stress was increased by 100 kp/cm² (10 N/mm²) per minute until the displacement rate was 1 mm/min. The rate of loading was then gradually reduced so that the rate of displacement was maintained at a constant level of 1 mm/min.

The first part of the displacement curves, fig. 3, was determined by means of dial indicators, fig. 4. These measurements served as a basis for the determination of the displacement modulus K and the proportional limit. The complete displacement curve was continuously recorded by mechanical means, fig. 5.

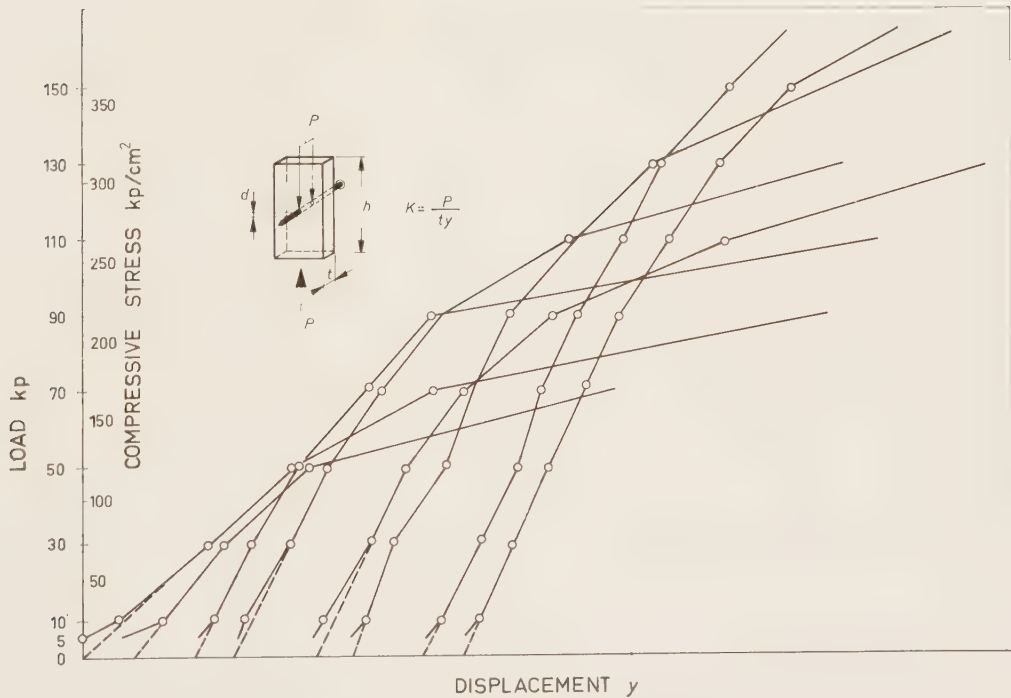


FIG. 3. Examples of displacement curves according to measurements with dial indicators. The displacement was also continuously recorded, cf. fig. 5. Dimensions of test specimens, see table 3.

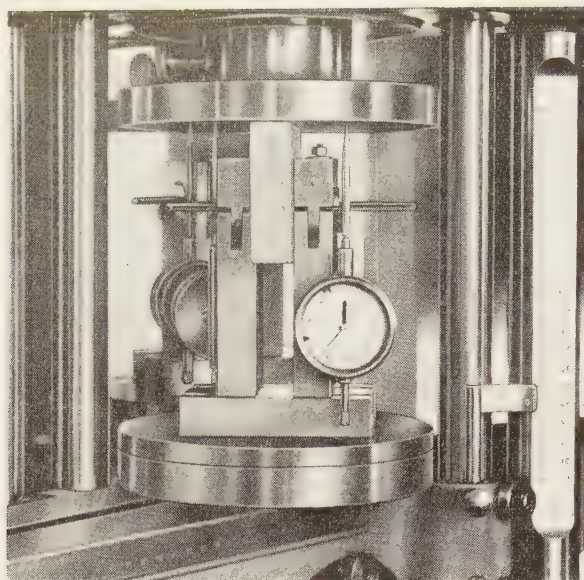


FIG. 4. Apparatus used for displacement tests. Force perpendicular to the grain.

In the figure, the load values have been indicated which were particularly studied, namely

P_p : Force at the proportional limit, defined as the force causing a displacement exceeding the displace-

ment corresponding to the displacement modulus K by 0.01 mm.

$P_{0.02d}$: Force causing a displacement exceeding the displacement corresponding to K by 0.02 d .¹

P_2 : Force causing a total displacement of 2 mm. (Here, as in the case of the definition of the above forces, the small initial displacement, y_1 , occurring in as well fig. 3 as in fig. 5 and which, at least in part, is due to the measuring devices used, is not taken into account.)

P_F : Force causing "flow". Since real flow (horizontal curve) rarely occurs, flow was assumed to occur when the force did not increase by more than 10 % during 1 minute, i. e. at a displacement of 1 mm.

P_{max} : The highest force that could be applied without causing a rate of displacement exceeding 1 mm/min.

¹ The reason why this load was determined was a German proposal (Fahlbusch, 1949) that the working load on bolted joints should be based on the load causing a permanent displacement in the joint of the magnitude 0.04 d , thus as a rule 0.02 d in the timber members on either side of the juncture.

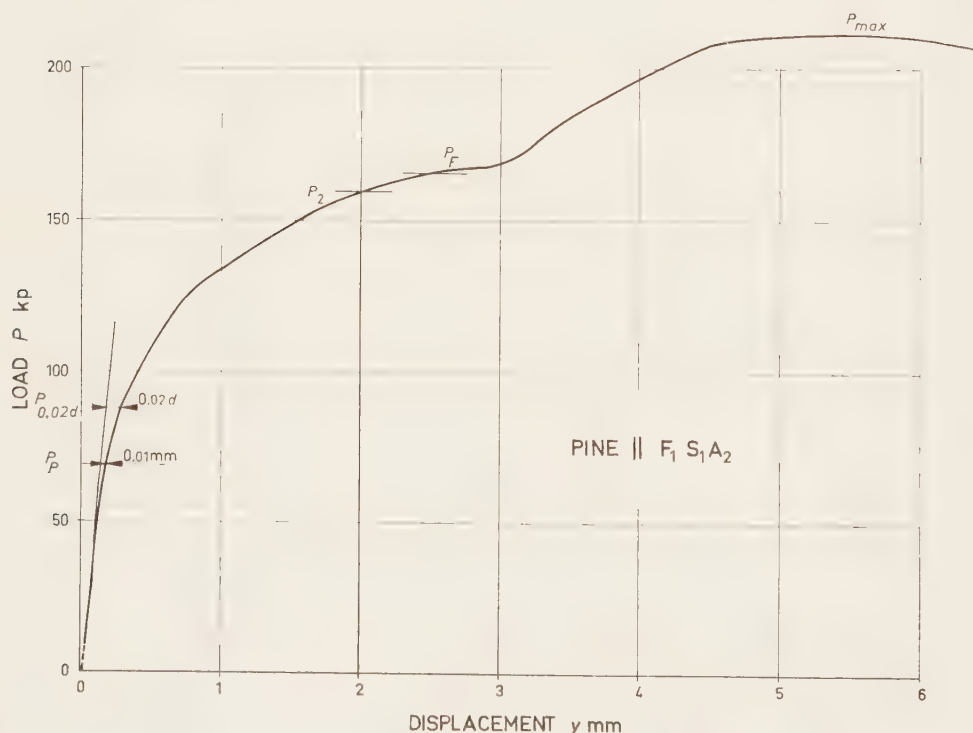


FIG. 5. Load-slip curve. K , the displacement modulus, indicates the slope of the first part of the curve, i. e. the relation between the force per unit length of the nail and the displacement. P_p load at proportional limit (the load that gives a displacement exceeding the displacement corresponding to K by 1/100 mm), $P_{0.02d}$ = load causing a deviation from the proportional limit corresponding to K by 2/100 of the nail diameter, P_{max} ultimate load.

TABLE 4. Number of displacement tests, stage 2.
Pine loaded || to the grain. Grooved wire nails (S_2). Constant moisture content (F_1).

Density level	Nails A_1 ($d = 2.2$)			Nails A_2 ($d = 3.2$)			Nails A_3 ($d = 5.0$)		
	No. of tests	$r_{0\mu}$ g/cm ³	u ‰	No. of tests	$r_{0\mu}$ g/cm ³	u ‰	No. of tests	$r_{0\mu}$ g/cm ³	u ‰
R 11				10	0.34	11.1			
R 21	10	0.39	11.3	10	0.34	11.4	5	0.37	11.6
R 22	10	0.43	11.1	10	0.42	10.8	5	0.42	11.0
R 31	10	0.45	11.4	10	0.43	11.3	5	0.44	10.9
R 32	10	0.47	11.3	10	0.46	11.3	5	0.47	10.9
R 41	10	0.49	10.9	10	0.47	11.4	5	0.49	11.3
R 42	10	0.54	10.6	10	0.52	11.2	5	0.54	10.6
R 51				10	0.60	10.1			
Number of tests	60	Average 11.1		80	Average 11.1		30	Average 11.1	

When the test results were available, it was found that, in the case of displacement parallel to the grain, the difference between P_2 and P_F was so small that it was deemed meaningless, in view of the deviation, to distinguish between these forces. The force causing 2 mm displacement parallel to the grain has therefore been regarded as “flow limit” and has been used as a basis for determining the strength parallel to the grain. With the chosen rate of displacement (1 mm/min), it was of course possible to obtain higher compression in certain cases (fig. 5). However, these maximum values were either insignificantly above the flow limit, or obtained after so great a displacement that they cannot be used as a basis for practical calculations of the strength of nailed joints.

The displacement tests belonging to stage 1 were made in 1953. Stage 2 was carried out six years later in connection with long-duration tests of nailed joints. The extent of these tests is shown in table 4.

Results

The results according to figs. 6—18 refer to nailing without prebored holes. Each timber species, fibre direction and the three characteristic moisture content levels are dealt with separately and the results in each case demonstrated by three graphs. The mean curve in the left-hand graph shows the relation between the dependent variable and the shape of the nail section (summed over A and R). The graph in the middle shows the influence of the nail diameter (summed over S and R). The mean curve in the right-hand graph shows the influence of the density on the dependent variable (summed over S

and A). In the graphs referring to the displacement modulus K , three separate curves are given in the left-hand graph (summed over R, S and A, respectively) but otherwise separate curves are only given when the analysis of variance showed (95 % probability) interaction.

An example is given in table 5 and the corresponding figure 11 a, moisture content level F_1 . The mean strength is 380 kp/cm² (38 N/mm²). The standard deviation (variance) can be divided into three components depending on S, A and R, respectively, and the interaction S·A, S·R, A·R and S·A·R and a residual deviation that cannot be directly ascribed to the influence of the independent variables. In the example, the mean strength values obtained from the tests with 2.3, 3.2 and 5.0 mm nails deviated very little from one another. The variance components depending on A and on S·A·R, respectively, were not significant (probability less than 95 %) and have consequently been included in the residual deviation in table 5. In fig. 11 a, the broken curve shows the mean variation of the strength in relation to the nail diameter. However, the interaction S·A is significant. This indicates that the influence of the nail diameter on the strength

TABLE 5. Modified table of variance (fig. 11 a; F_1).

Source of variance	Degree of freedom	Sum of squares	Variance
S	2	132,984	66,095
R	2	160,189	80,095
S · R	4	32,916	8,229
S · A	4	48,048	12,012
Residual deviation	41	102,087	2,490
Total		476,224	

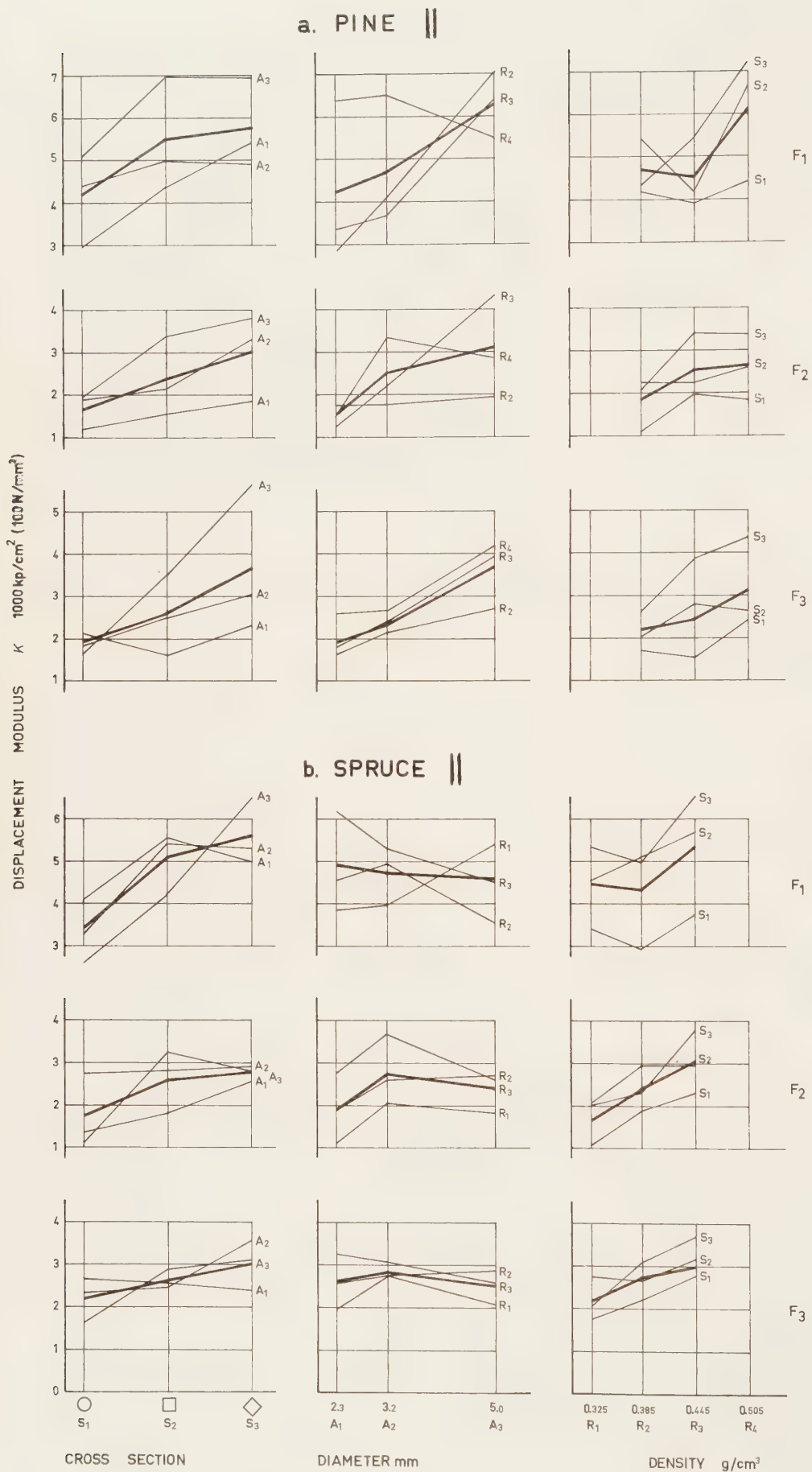
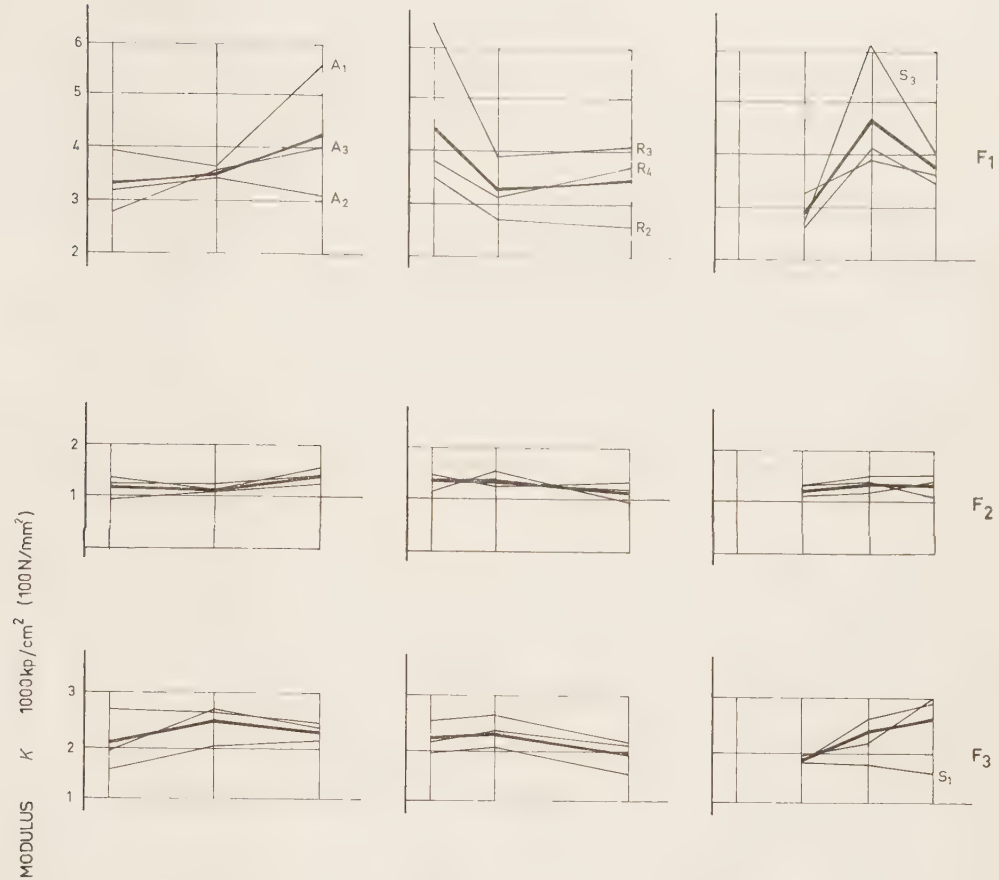


FIG. 6. Influence of the variables on the displacement modulus. The figure contains four graphs, a—d, showing the values for pine and spruce at force parallel and perpendicular to the grain, respectively. Other variables than timber species and directions of force are:

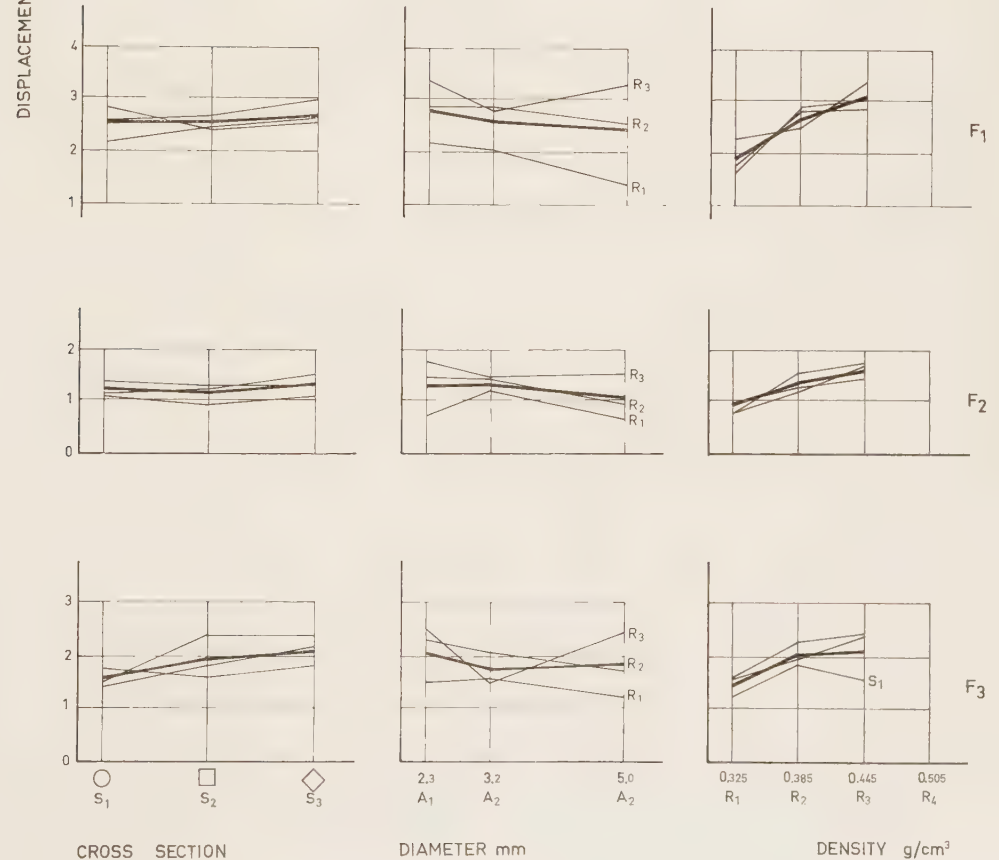
Sections of nails: S_1 round, S_2 grooved square, S_3 grooved square driven diagonally to the grain.

Diameter of the nails: A_1 2.3 mm (round 2.1), A_2 3.2 mm (round 3.5), A_3 5 mm.

c. PINE $\perp$



d. SPRUCE $\perp$



Dry density of the timber: R_1 — R_5 density levels from r_{0H} about 0.35 g/cm³ to 0.55 g/cm³.

Moisture content at nailing and testing: F_1 moisture content in both cases 12 %, F_2 moisture content in both cases 30 %, F_3 moisture content at nailing 30 % and at testing 12 %. The influence of the variables on the displacement modulus is shown without regard to the statistical significance of the variations.

varies depending on the type of the section of the nail. The three separate curves for S_1 , S_2 and S_3 are consequently given in the graph in the middle. (It is also possible to say that the influence that the type of the section has on the strength varies with the nail diameter and to include the curves in the left-hand graph. However, where there is interaction, the separate curves have always been referred to one of two possible graphs.) That the three separate curves are represented by a continuous line does not mean that each individual type of section gave a strength significantly depending on the nail diameter. The number of test results constituting a single curve was too small to justify a test for the significance of correlation.

The displacement modulus K

The displacement modulus K is given in the graphs, figs. 6 a—d. The deviation of the K values is very great, and the results must consequently be judged with caution. The deviation in part depends on the fact that each nail was tested separately and, to some extent, also on the difficulty of objectively determining the slope of the first part of the load-slip curve (cf. fig. 3). In some cases a comparatively great initial displacement was recorded, which rarely occurs in ordinary nailed joints with several nails. This initial displacement, part of which may depend on factors which are not related to the movement of the nail in the timber, has not been included in the proportionality modulus K .

As regards the influence on the displacement modulus exerted by the type of the nail section, figs. 6a,b clearly show that round nails gave a lower displacement modulus than grooved wire nails in both pine and spruce. If the orientation of the section of the grooved wire nails in relation to the grain has any influence on K is more doubtful. However, the results may be said to imply that a diagonally placed nail gives a displacement that is equal to or smaller than the displacement when the ridges of the nail are parallel to the grain. In practice, this means that we are on the safe side if nail is assumed to be in the latter position (S_2). This also applies to forces perpendicular to the grain (figs. 6c,d).

As regards the influence of the nail diameter on the displacement modulus, pine and spruce surprisingly give different results. In spruce the displacement

modulus K may, according to fig. 6 b, regarded as independent of the nail diameter, 4500 kp/cm² (450 N/mm²) in dry timber and 2500 kp/cm² (250 N/mm²) in green timber (irrespective of whether the timber was green or dry when nailed). In pine the displacement modulus, according to fig. 6a, at each of the three moisture content levels, depended on the nail diameter (d in mm) approximately according to

$$K_F = C_F(0.2d + 0.3) \quad (3)$$

The value of the coefficient C_F may be assumed to be 5000 kp/cm² (500 N/mm²) for dry timber (F_1) and half this value, i. e. 2500 kp/cm² (250 N/mm²) if the timber is green (F_2) or was so at the time of nailing (F_3).

The results, fig. 6c,d, show that neither the section (S) nor the diameter of the nail (A) had any significant influence on the displacement modulus perpendicular to the grain. The only deviating results are the comparatively high values obtained for diagonally placed thin nails in dry pine (S_3 , A_1 , F_1). An increasing value of K with increasing density was observed for both pine and spruce parallel as well as perpendicular to the grain. For pine parallel to the grain, the results can be expressed by

$$K_F = C_F(0.2d + 0.3)(3r_{0u} - 0.33) \quad (4)$$

where d is the diameter in mm and r_{0u} the dry density in g/cm³. Thus, (3) is applicable when $r_{0u} = 0.445$, corresponding to the density level R_3 , which is the mean density in tests presented in fig. 6a.

In the tests on spruce, fig. 6 b, R_2 was the mean density. However, if the displacement modulus at the density level R_3 is introduced as the value of the modulus for spruce, C_G , the dependence on the density of K is approximately the same for spruce as for pine:

$$K_G = C_G(3r_{0u} - 0.33) \quad (5)$$

For spruce there is no equivalent to the diameter dependent factor in (4). As fig. 7 shows, the factor for the influence of the density on the displacement modulus given for the force parallel to the grain can also be used perpendicular to the grain.

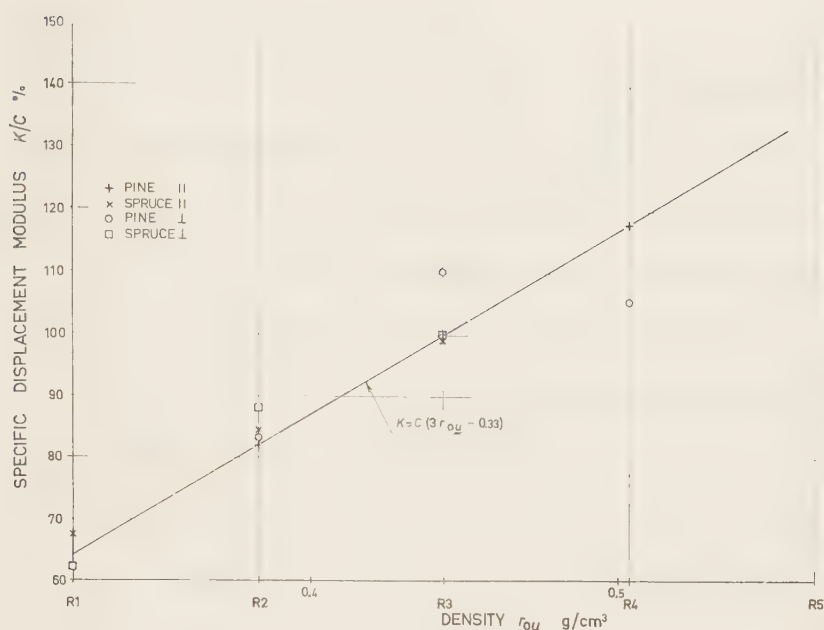


FIG. 7. Influence of the density on the displacement modulus. The displacement modulus (K) is given in per cent of the value at the density level R_3 , i. e. 0.445 g/cm^3 . The values are means for the three moisture content levels.

The dependence on the diameter, which according to fig. 6 a occurs in pine parallel to the grain, makes it difficult to summarize the K values. The picture is further complicated by the tendency of the diameter dependence to disappear, or even to reverse sign, as the density increases. (The K values at the moisture level F_1 and the density level R_4 are lower for the diameter 5.0 mm than for the thinner nails. Tests—here not dealt with—on timber with the density level R_5 gave the same result.) However, the material is not large enough to permit of reliable conclusions as to the interaction $A \cdot R$. Other results were also obtained during the second stage of the investigation, which inter alia comprised precisely the case pine parallel to the grain, dry timber and grooved nails (S_2). The K values obtained are given in fig. 8, which clearly shows that the deviation was considerable. Obviously, it is not possible to arrive at a diameter influence on K on the basis of this material. Also the influence of the density on K is, on the whole, obscured by the deviation. However, the density factor used in (4) and (5) is quite acceptable also for describing the results obtained during the second stage of the investigation according to fig. 8. Surprisingly, stage 2 without exception resulted in lower K values than stage 1. A structural difference not observed in the timber used for the two stages may have resulted in diffe-

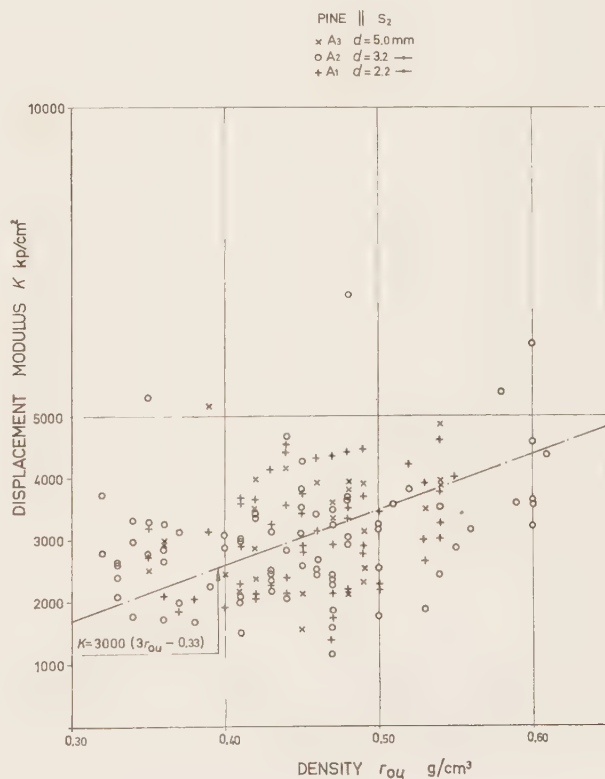


FIG. 8. The displacement modulus K —wire nail in pine, force parallel to the grain—according to results obtained at stage 2. The dash-and-dot line shows the non-diameter-dependent displacement modulus assumed on the basis of the results obtained at stage 1.

rent K values at the same density. On the other hand, a certain subjectivity in determining the slope of the load and deformation curves cannot be excluded. Different persons were responsible for the two stages and the recording methods were not identical.

It is probably possible, by considerably more comprehensive tests, to confirm that the displacement modulus has different values in pine and spruce both parallel and perpendicular to the grain. Nevertheless, against the background of the results described here, it seems wise to adopt a common value for pine and spruce for the practical calculation of the initial displacement in nailed joints, a value that may moreover be used irrespective of the angle between the force and the grain. For normal air-seasoned timber, this value may be set at $K = 3000$ kp/cm² (300 N/mm²), or if the deviation from normal density, g/cm³, is to be taken into account

$$K = 3000 (3r_{0u} - 0.33) \quad (6)$$

For green timber, with or without subsequent drying, it is reasonably in accordance with the results to set the displacement modulus K at half this value. (The value of the displacement modulus K for dry timber is obtained by multiplying the strength of the prisms by 6.6, cf. figs. 21 a,b.)

Proportional limit

The proportional limit of the stress under the nails, σ_p , was, as fig. 9 shows, not significantly influenced by the section and diameter of the nails or the timber species and the direction of the force. For dry timber the proportional limit might, according to fig. 9, very well be written

$$\sigma_p = 150 (3r_{0u} - 0.33) \text{ kp/cm}^2 (0.1 \text{ N/mm}^2) \quad (7)$$

The tests, stage 2, resulted in a proportional limit about 30 % higher than at stage 1. (It is observed that if the slope of the first part of the load—slip curve is given too high a value, corresponding to a low K value, cf. fig. 8, the result will be a higher stress value at the proportional limit.)

No difference was observed between the proportional limit stress in constantly dry timber (F_1) and the cases in which the timber was green nailed but dry when tested (F_3). In green timber (F_2) the proportional limit was lower. In practice it should

perhaps be assumed that the proportional limit is 50 % lower in green than in dry timber, i. e. that it falls at the same rate as the displacement modulus K .

Compressive strength under nails

Since a nailed structure can stand smaller or greater displacements in the joints before failure should be considered to have occurred, opinions may differ as to what is to be regarded as failure load and consequently also ultimate stress in nailed joints, i. e. the strength under the nails. In Sweden, the working load on nailed joints as well as bolted joints is mainly based on the load that will cause flow in the joint. This means that the strength is defined by a value comparatively high on the load—slip curve and which approaches the maximum value obtained by short-duration tests. As regards bolted joints, working stresses in the USA have been based on the proportional limit stress. In other cases the practical strength limit has been set at a certain displacement in the joint. It has inter alia been proposed (Fahlbusch, 1949) that the working stress in joints (particularly bolted joints) should be based on the load causing a certain permanent displacement between the members, or to be exact $0.04 d$, where d is the diameter of the fastener. This would imply that the permanent displacement of the fastener in the timber is maximized at $0.02 d$.

In order actually to determine if a deformation in a nailed or bolted joint is irrecoverable, a fairly long time of recovery is necessary. In any case, where the displacement of the nail is parallel to the grain, the recovery after unloading, in addition to the recovery corresponding to the displacement modulus, is slight. The stress $\sigma_{0.02d}$ which, according to the above, limits the irrecoverable deformation, is here defined as the stress resulting in a displacement exceeding the displacement calculated on the basis of K by $0.02 d$. The total displacement can thus be expressed as

$$y_{0.02d} = (\sigma_{0.02d}/K + 0.02) d \quad (8)$$

In reality the stress thus defined is not much above the proportional limit and we may expect a similar relation to the different variables as in the case of the proportional limit stress. A comparison between the graphs, figs. 9 and 10, also shows very similar curves. However, the value of $\sigma_{0.02d}$ without exception exceeds the proportional limit by 1/3, i. e. $\sigma_{0.02d}$ is obtained if we in (7) replace the coefficient 150 by 200 (for green timber by 100).

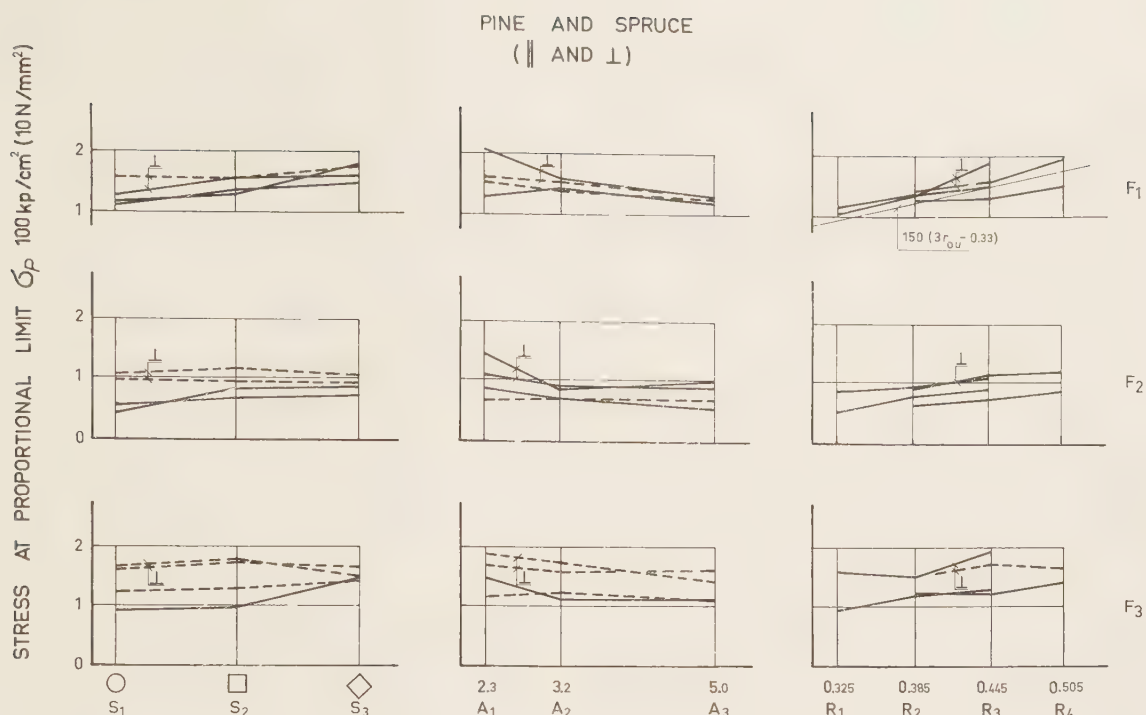


FIG. 9. Mean values of stress at proportional limit. Broken curve = Influence of independent variable not significant at 95 % level.

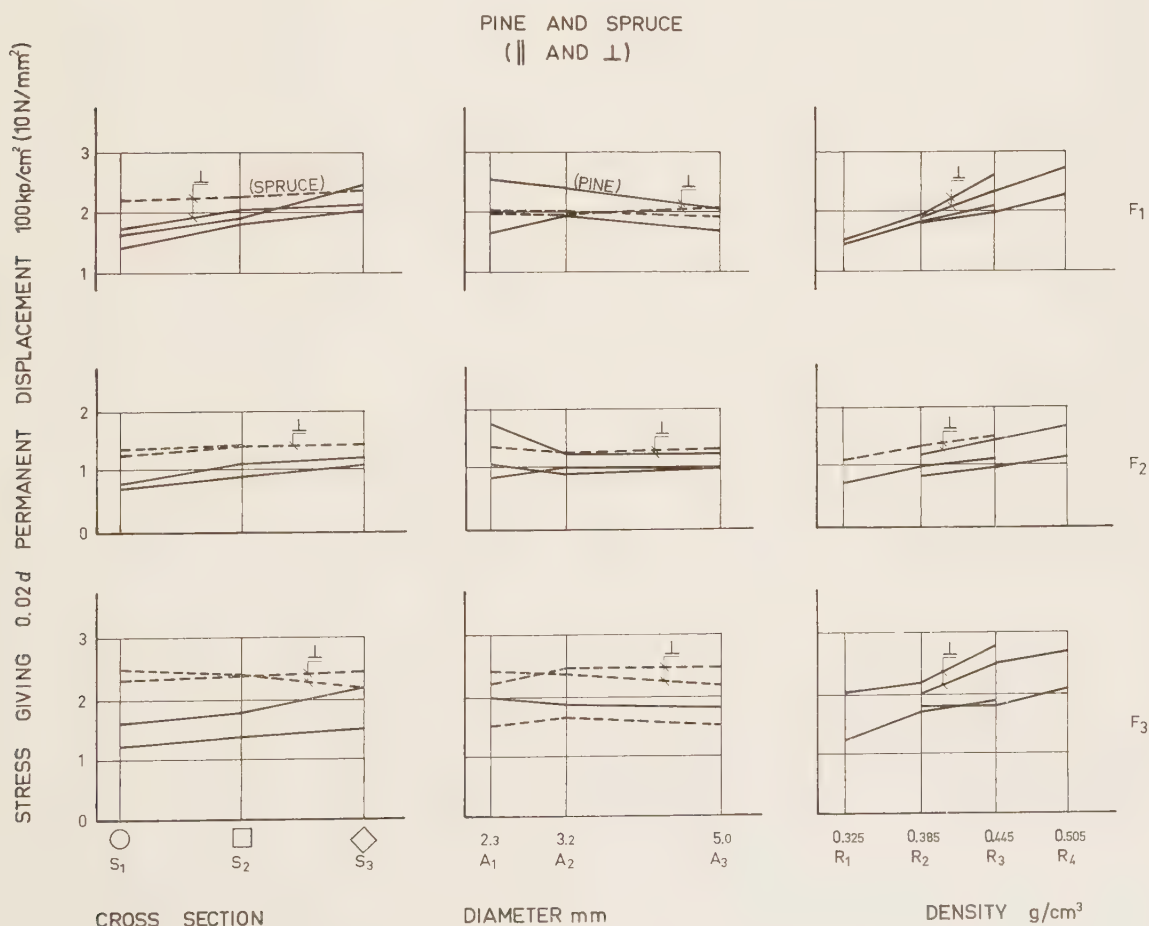


FIG. 10. Stress giving a permanent displacement of 0.02 times the nail diameter. Cf. fig. 9.

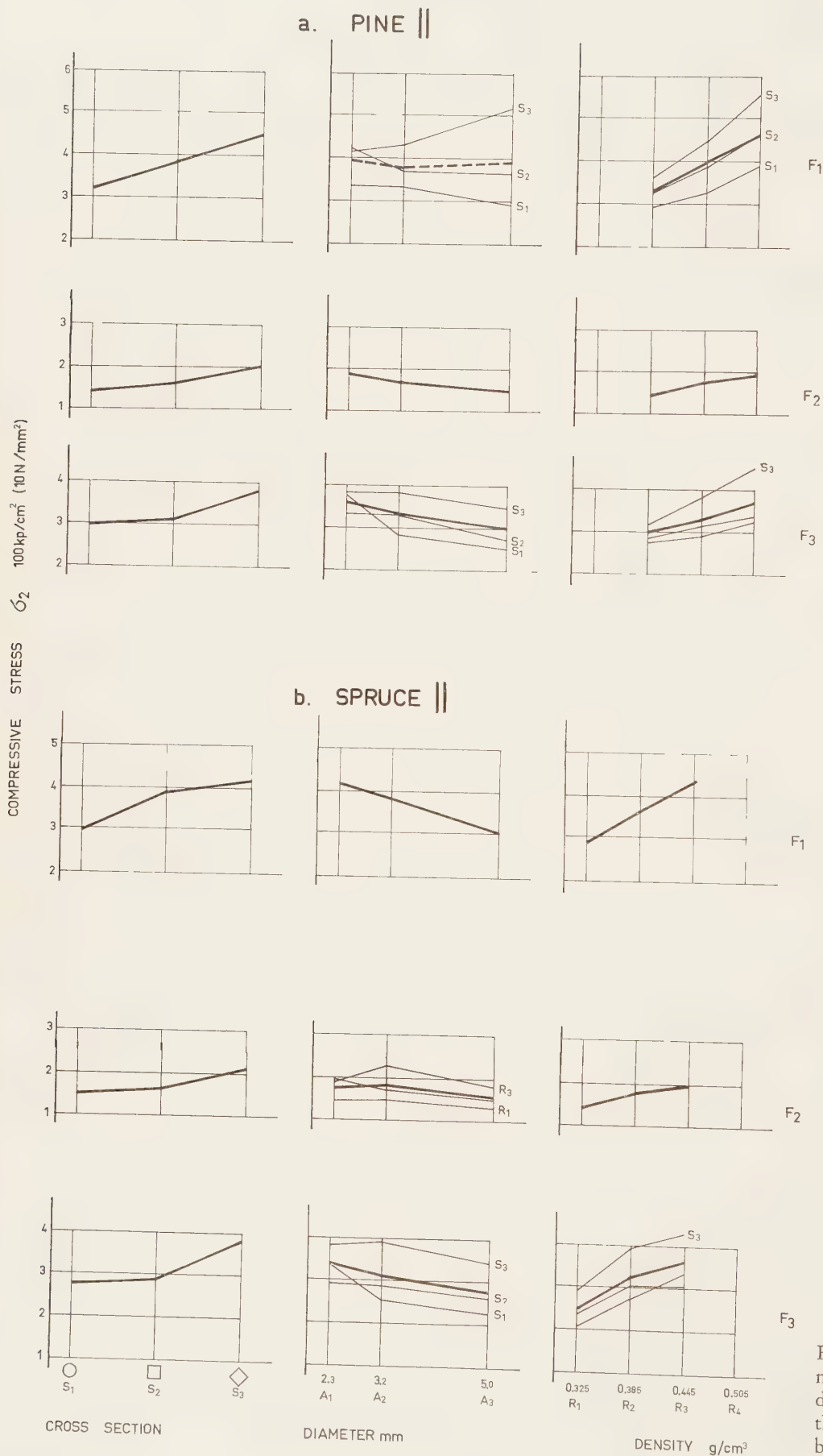
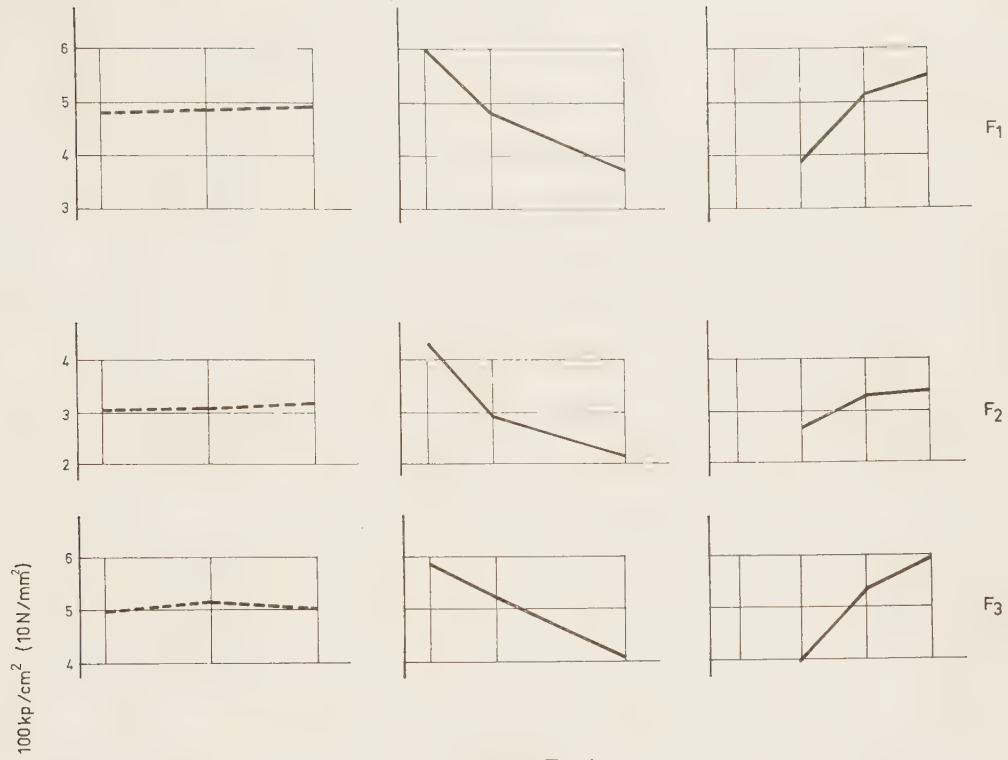
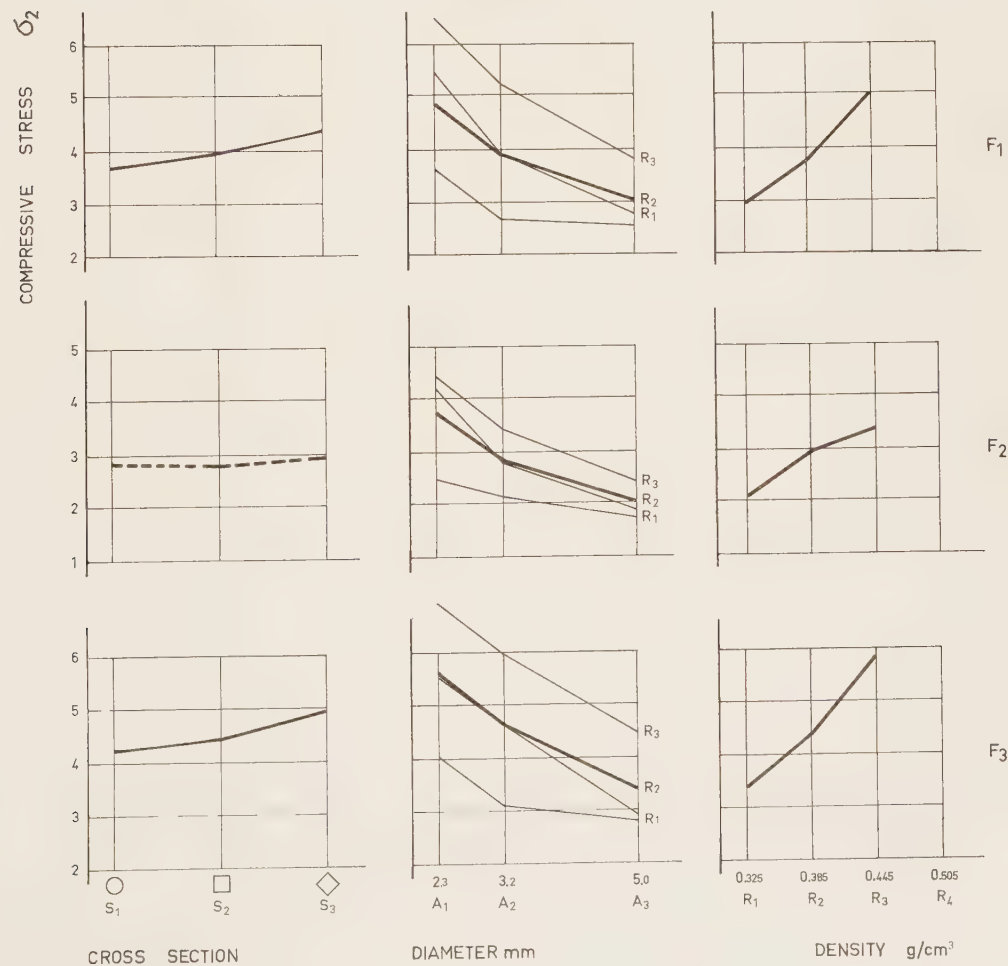


FIG. 11. Stress giving 2 mm displacement. Where the influence of the independent variable is not significant, the mean curve is represented by a broken line. In case of significant in-

c. PINE ⊥



d. SPRUCE ⊥



teraction between the independent variables, there are, in addition to the mean curve, subcurves. See also text to fig. 6.

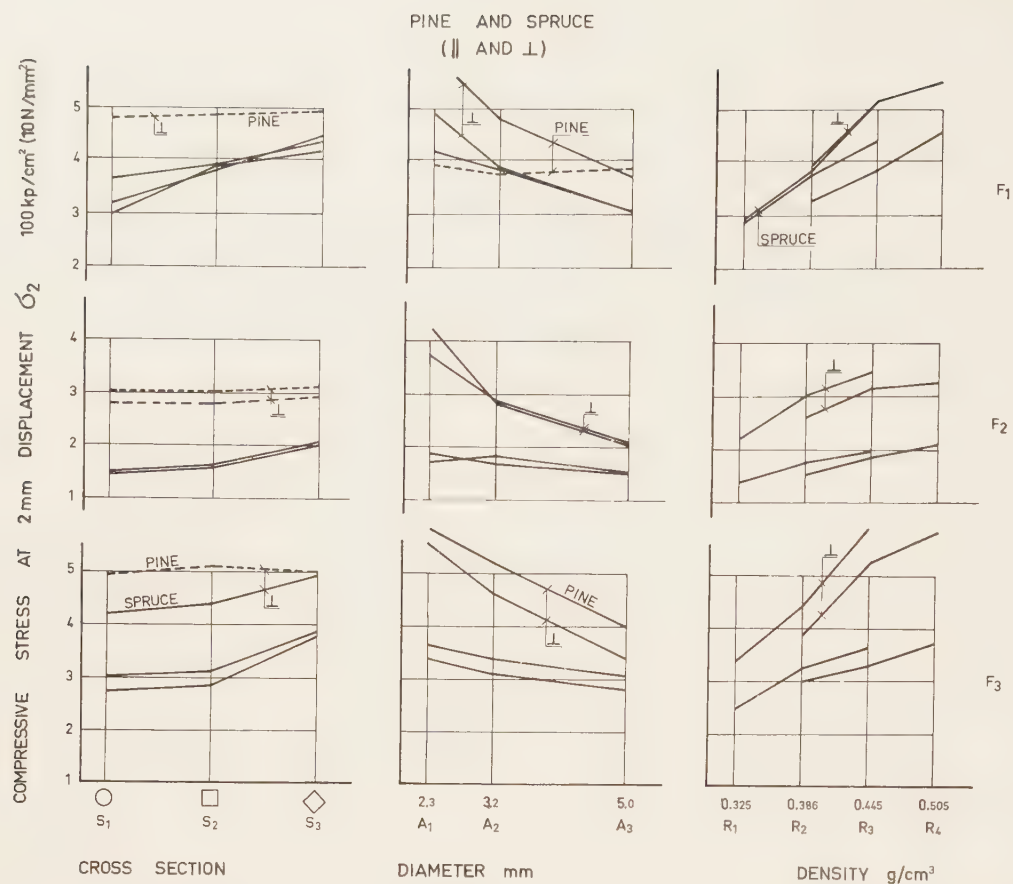


FIG. 12. Mean values of stress giving 2 mm displacement. The mean curves are brought together, irrespective of possible interactions.

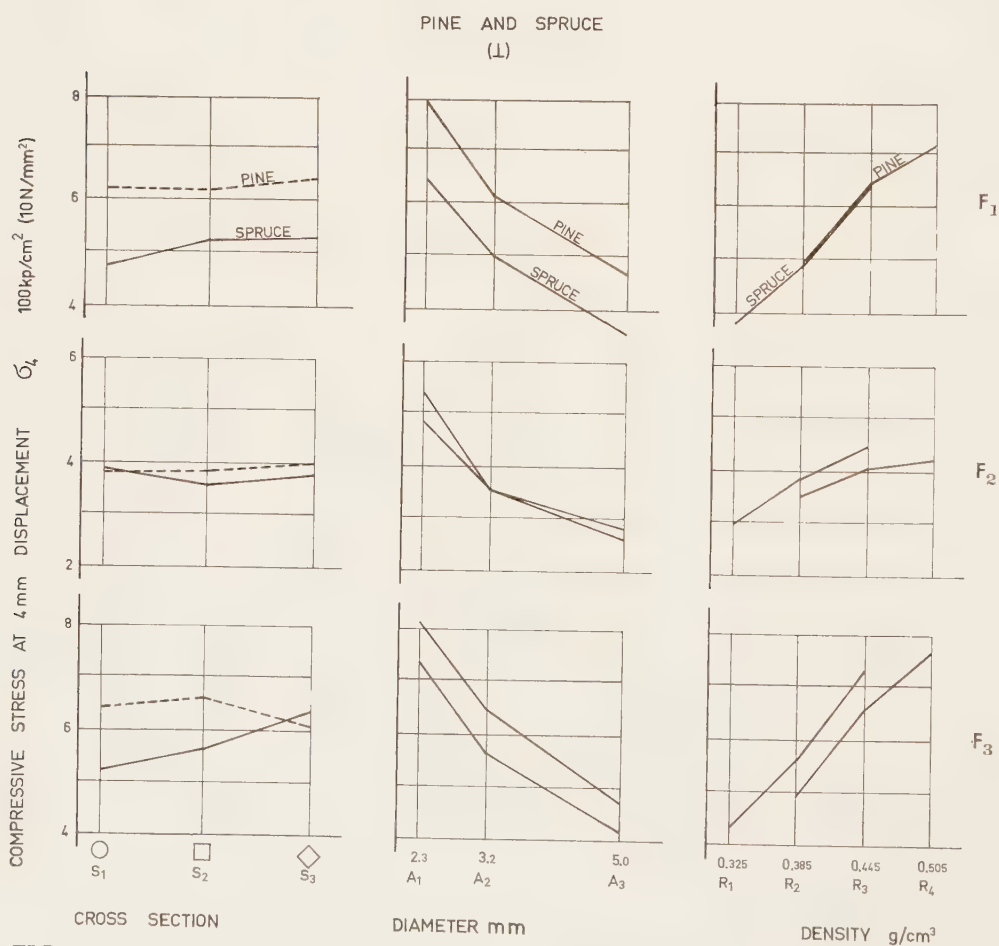


FIG. 13. Mean values of stress giving 4 mm displacement perpendicular to the grain.

It was found that the flow value of the stress, as defined above, parallel to the grain closely corresponded to the stress measured at 2 mm displacement. Perpendicular to the grain, flow was in many cases not obtained at all or only at a displacement of several mm. Therefore, the flow value of the stress was replaced by the stress corresponding to 4 mm displacement (reproduced in fig. 13). However, even a displacement of this magnitude is of minor practical importance. Thus, in the following, the stress resulting in 2 mm displacement, denoted σ_2 , is referred to as "strength" parallel as well as perpendicular to the grain.

The strength parallel to the grain is given in figs. 11a,b (perpendicular to the grain in figs. 11c,d, mean values for both directions in fig. 12). On an average, the round nails resulted in lower strength than the grooved wire nails. Furthermore, there was a slight increase in strength when the grooved wire nails were placed diagonal to the grain. Practically without exception, the results also show a slightly reduced strength as the nail diameter increases. In pine and at the moisture content level F_1 , the dependence on the diameter is not clear. Yet, in this case the results obtained during stage 2, fig. 14, might serve as an indication. As regards the interactions, $S \cdot A$ and $S \cdot R$, it is observed that the mean result on the whole agrees with the result for grooved wire nails (S_2).

For dry spruce and pine we may write

$$\sigma_2 \parallel = 380 (-0.07d + 1.25)(3r_{ou} - 0.33) \quad (9)$$

where 380 is the strength in kp/cm^2 at a diameter, d , of 3.5 mm and a density, r_{ou} , of 0.455 g/cm^3 . The dependence on the diameter is the mean result from the three moisture levels in stage 1 and smaller than according to stage 2, fig. 15.

The influence of the diameter on the stress causing 2 mm displacement is very marked at compression perpendicular to the grain. The values in fig. 16 are mean values for all moisture content levels. If the dependence on the diameter is assumed to be rectilinear, it can—with d in mm—be expressed according to

$$\sigma_2 \perp = 450 (-0.16d + 1.56)(3r_{ou} - 0.33) \quad (10)$$

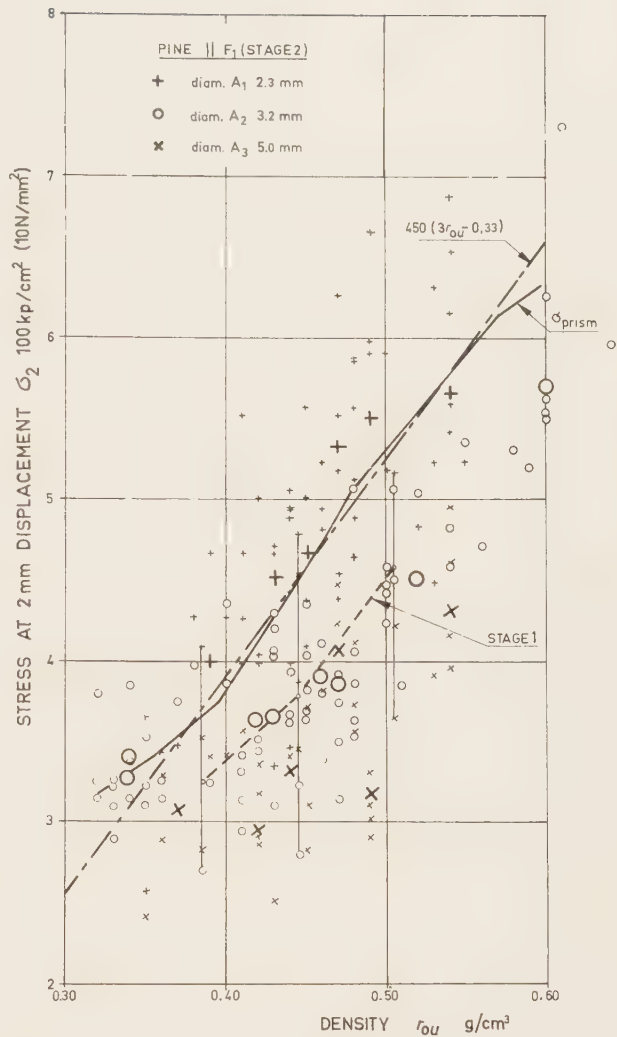


FIG. 14. Strength under the nails (stress causing 2 mm displacement) according to stage 2 (grooved wire nail, dry pine, force parallel to the grain). Broken line indicates mean results from corresponding tests, stage 1 (individual values indicated along the respective vertical lines). The S-shaped curve shows the strength of the timber parallel to the grain according to fig. 20.

The results, however, indicate that the dependence on the diameter diminishes at higher diametrical values. (It should be observed that part of the deviation from the straight line is due to the fact that the values refer to the diameter of the grooved wire nails. In consideration of the fact that the round nails had a slightly deviating diameter, the values may be assumed to be 2.2 and 3.3 instead of 2.3 and 3.2 mm). However, the dependence on the diameter is not quite so great as to correspond to the case that the displacement should be directly proportional to the compression per unit length of the nail (broken hyperbola in fig. 16).

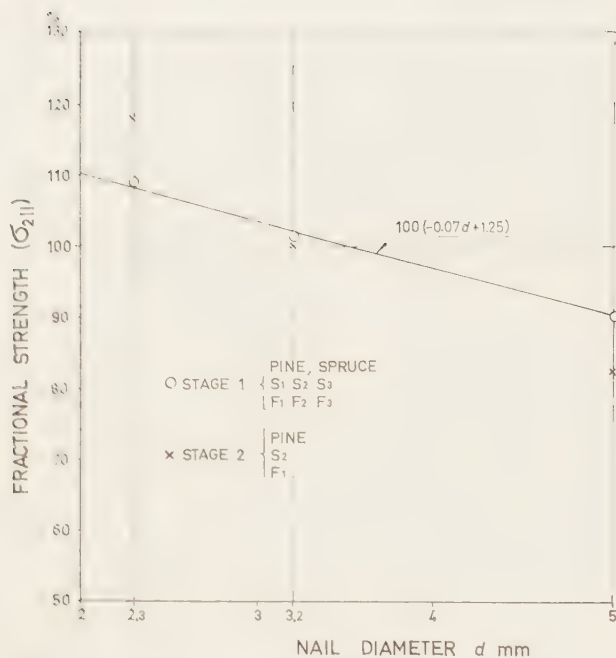


FIG. 15. Diameter dependence of the strength parallel to the grain. The strength is expressed in per cent of the strength when the nail diameter is 3.5 mm. The line represents the results according to stage 1, fig. 11 a and b. The results according to stage 2 (×) are calculated on the basis of the values in fig. 14.

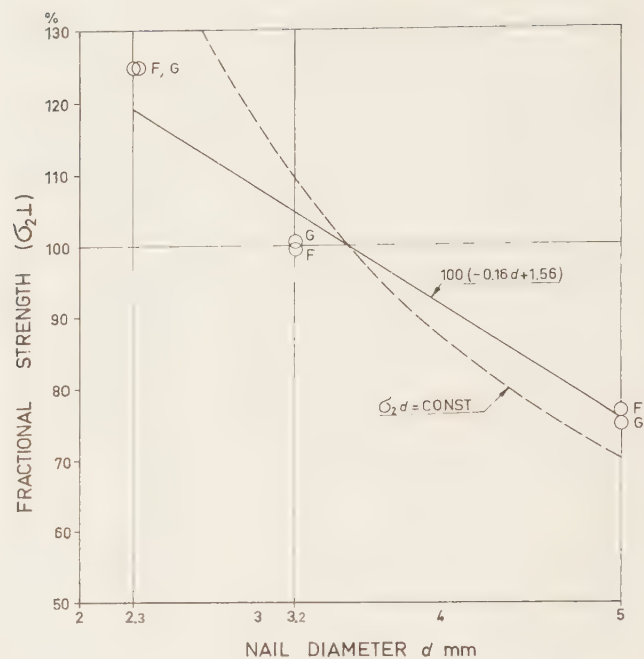


FIG. 16. Diameter dependence of the strength perpendicular to the grain in pine (F) and spruce (G). The strength is expressed in per cent of the value for the nail diameter 3.5 mm (calculated on the basis of an assumed linear diameter dependence). Tabulation of results from fig. 11 e and d. The diameter dependence is not quite so great as to correspond to a constant force per unit length of the nail (broken curve).

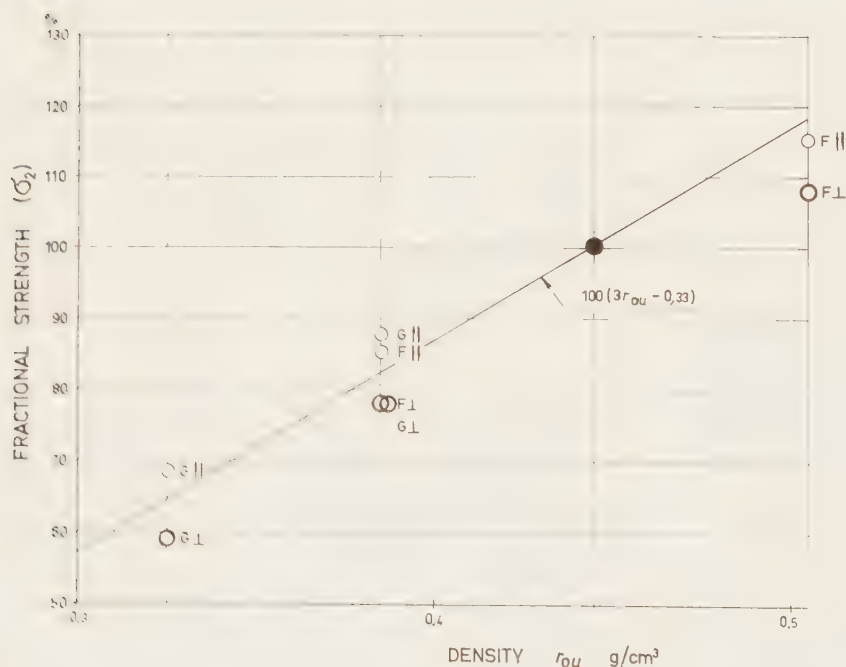


FIG. 17. Density dependence of the strength according to stage 1. The values are given in per cent of the strength at a density of 0.445 g/cm³. The straight line represents the density dependence established for the other parts of the load-slip curve as well as the density dependence of the prism strength. F = pine, G = spruce, || = parallel, ⊥ = perpendicular to the fibres.

The influence of the density on the strength (σ_2), irrespective of timber species and direction of force, approximately follows the regression line common for the displacement modulus, the proportional limit stress and the prism strength of the timber parallel to the grain. This is shown in fig. 17, where all test results from stage 1 are summarized.

The basic strength value for dry pine and spruce parallel to the grain is, according to (9), 380 kp/cm² (38 N/mm²). For green timber the value obtained was about 190 kp/cm² (19 N/mm²), i. e. half the former value. Perpendicular to the grain, the stress resulting in 2 mm displacement is greater. A suitable common value for dry pine and spruce is 450 kp/cm² (45 N/mm²) and for green timber 300 kp/cm² (30 N/mm²). The basic values refer to the dry density 0.445 g/cm³ and must for other densities be multiplied by $(3r_{0N}-0.33)$. The values apply primarily to grooved wire nails. For round nails, the value should be reduced by 15 % parallel to the grain. (The fact that the strength is higher perpendicular to the grain than parallel to it can, for practical reasons, not generally be utilized when nailed joints are designed.)

The strength values from stage 1 are summarized in fig. 12. As regards the strength of pine joints—grooved wire nails (S_2)—parallel to the grain, the results for each of the three nail diameters, obtained at both stages, are given in fig. 18.

Stresses in relation to strength of prisms

The original intention was to analyse the stresses at the nail displacement tests expressed in relation to the strength of the control prisms. The dependence on the density would then have disappeared from the analysis. The residual deviation, however, proved to be comparatively great when this method was used and it was therefore dropped.

As the timber for the groups of joints was sampled at random within each level of density, the mean strength value of the control prisms, which were all tested dry (F_1) and parallel to the grain, could be expected to be the same in all groups, i. e. for pine and spruce respectively. This was checked by applying the same analysis of variance to the strength values of the prisms as the one used for the joints.

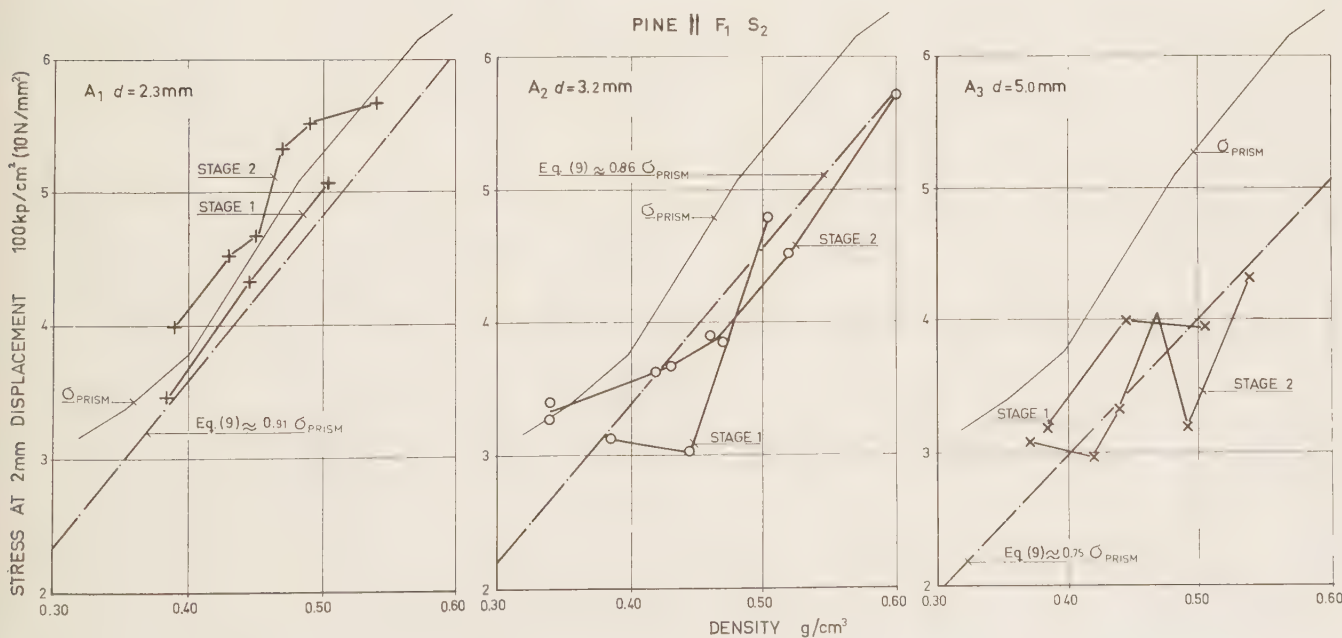


FIG. 18. Stress giving 2 mm displacement, according to eq. (9) (dash-and-dot lines) and according to the test results, compared with the strength of the control prisms.

PINE AND SPRUCE

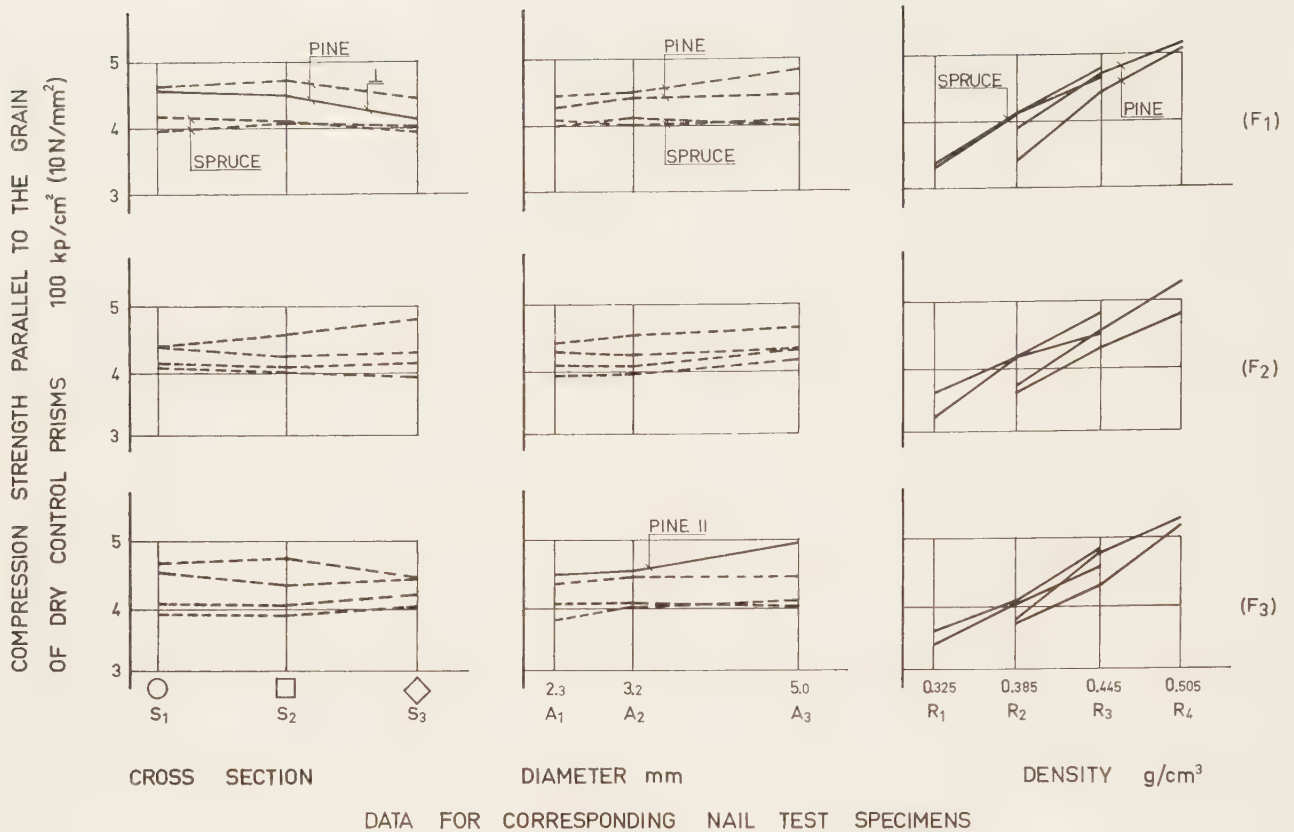


FIG. 19. Compressive strength of the timber parallel to the grain determined of control prisms. All prisms were tested at moisture content level F_1 . The independent variables refer to the corresponding displacement tests. The curves give an indication of the reliability of the results of the displacement tests, since the prism strength is to be significantly dependent (continuous curve) on the density only.

The result of this analysis is shown in fig. 19 in analogy with the results for the strength under the nails shown in fig. 12. Only in two cases the influence of S or A is significant at the 95 per cent level, indicating that the strength of the timber in the joints, in the combinations Pine $\perp$ S_3F_1 and Pine $\parallel$ A_3F_3 , was slightly higher than in the other joints. However, these deviation have no influence on the approximate formulas given for the strength properties of the joints.

Fig. 24, p. 32 shows the influence of the density on the prism strength according to about 500 compression tests on pine, stage 1. The line showing the correlation has been fitted to the data by means of least squares. This line, as well as the corresponding line

for spruce (from fig. 25), is compared with the general density dependence found at the nail displacement tests. The lines deviate only slightly. This means that there is a constant correlation between the stresses on the displacement curve of the nail and the strength of the timber parallel to the grain within the range of densities studied. The general relation between the different stresses at the displacement tests and the strength of the prism ($\sigma \parallel$) is evident if eq. (3) to (10) are compared with the lines in fig. 24. For dry pine under compression from grooved wire nails (S_2) parallel to the grain, the relation between the strength under the nail and the strength of the prism is shown in fig. 18. For the diameter 5 mm, the value is 0.75, which is in accordance with earlier results (Möller, 1951).

Nails and bolts in prebored holes

As has earlier been shown (Norén, 1955), the working stresses in nailed joints according to DIN 1052 correspond to a diameter dependence of the strength according to

$$\sigma_F = 0.7 \left(\frac{21}{14 + d} \right)^2 (\sigma_F)_{d = 3.5 \text{ mm}} \tag{11}$$

Furthermore, the influence of the diameter on the strength under bolts has been expressed by the author according to

$$\sigma_F = \frac{66}{60} d (\sigma_F)_{d = 6 \text{ mm}} \tag{12}$$

applicable to diameters between 6 and 24 mm.

The purpose of the tests with prebored holes was primarily to control the validity of (12), particularly for smaller diameters. The result of the tests on pine are found in fig. 20. The figure shows the flow limit, the proportional limit stress and the displacement modulus of dry and soaked timber (F_1 and F_2). The different curves refer to different density levels. The results do not indicate any interaction between the density and the nail diameter, i.e. the influence of the diameter is of the same nature, irrespective of the density. Similar results were obtained by testing spruce. The dependence on the density, summarized in fig. 21, is similar to the one found in the previously described tests without prebored holes (dot-and-dash lines).

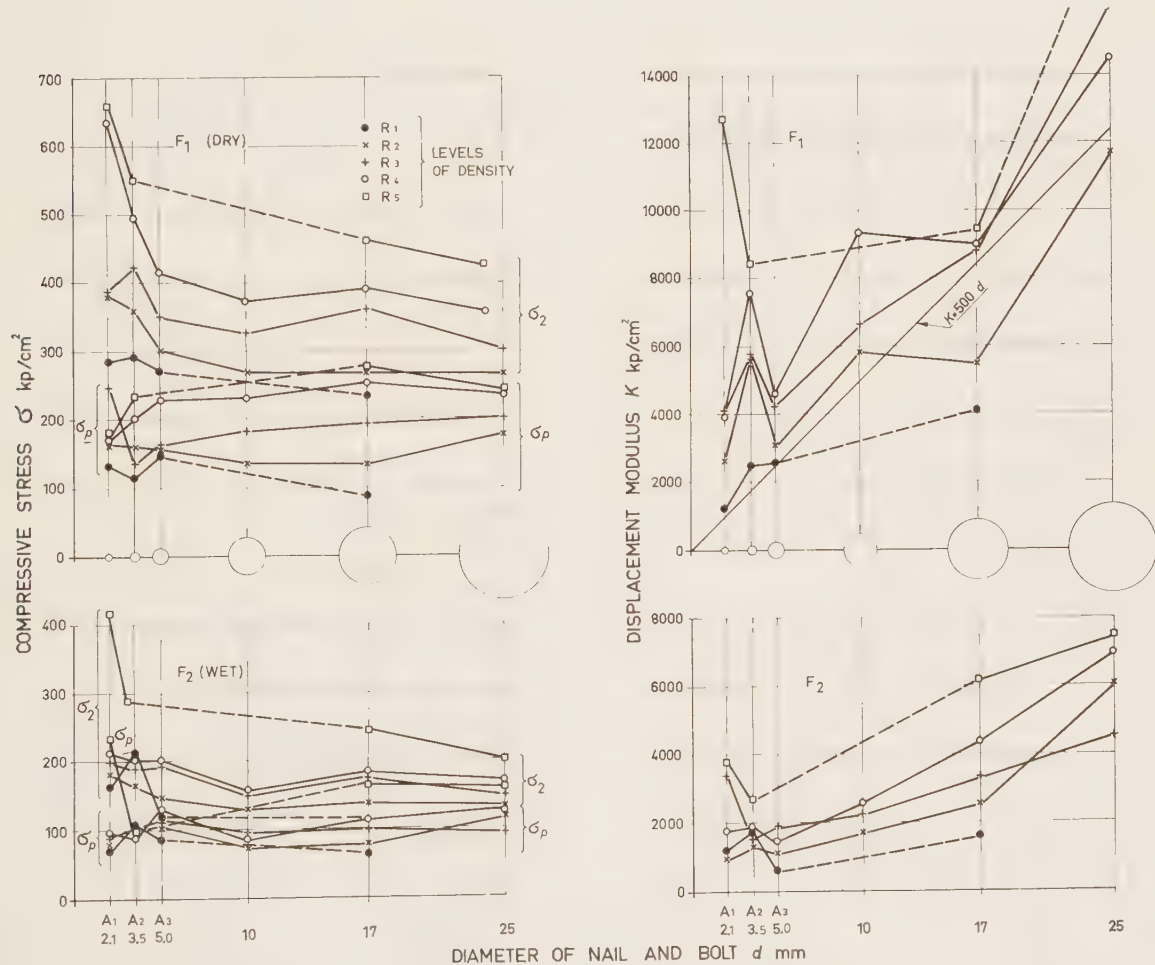


FIG. 20. The influence of the diameter on the displacement modulus K , the proportional limit stress σ_p , and the strength σ_2 (stress causing 2 mm displacement) under nails (round section) and bolts in dry pine parallel to the grain. Prebored holes were used in all cases. The curves represent different density levels R . Dry timber (F_1), soaked timber (F_2).

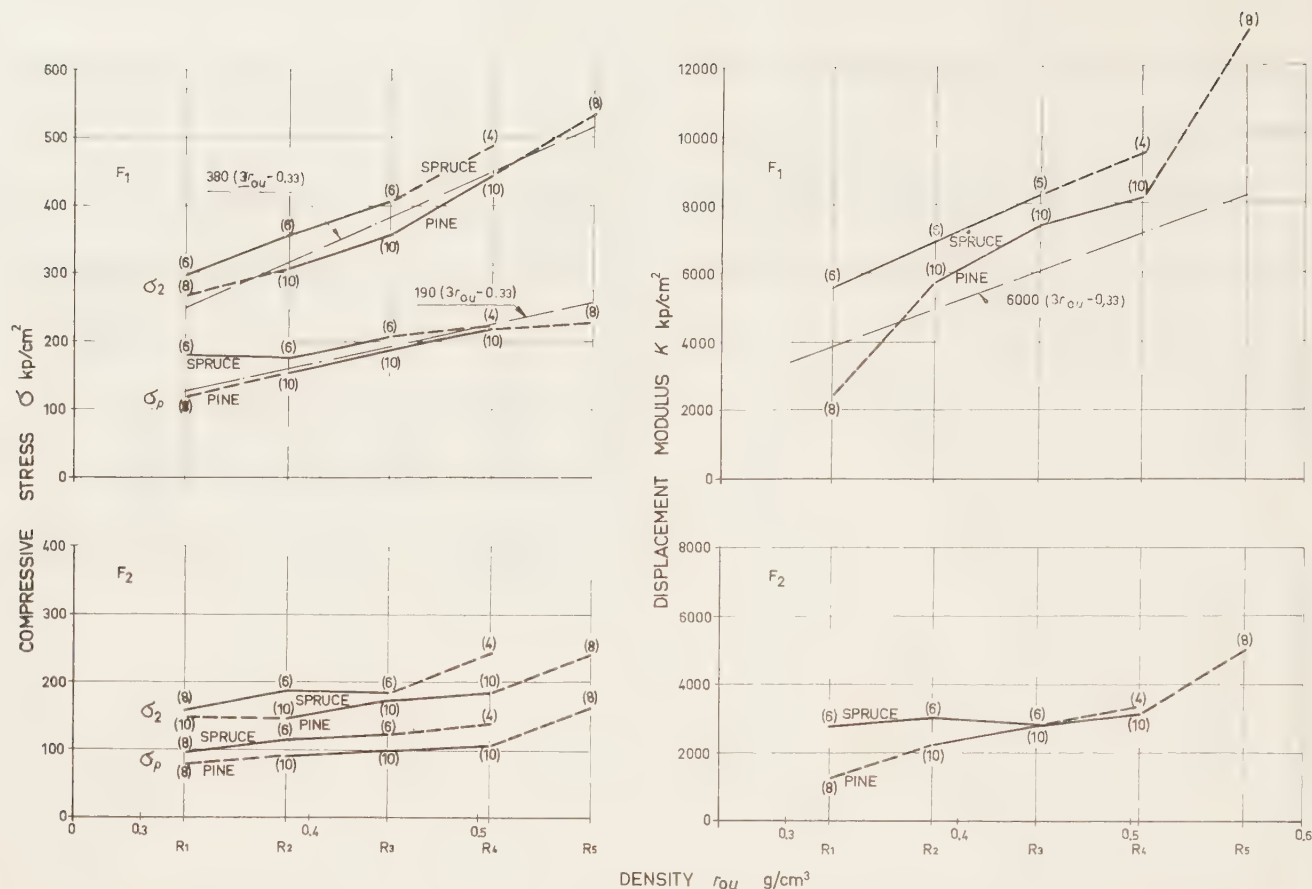


FIG. 21. Density dependence of the stress and displacement values of nails and bolts in prebored holes in pine (cf. fig. 20) and in spruce. Arithmetic means are used, in spite of the fact that the dependence on the diameter is non-linear. (Number of tests indicated by the values in brackets.) The density dependence agrees with the results obtained without prebored holes as well as with the density dependence of the prisms' strength (fig. 24, 25).

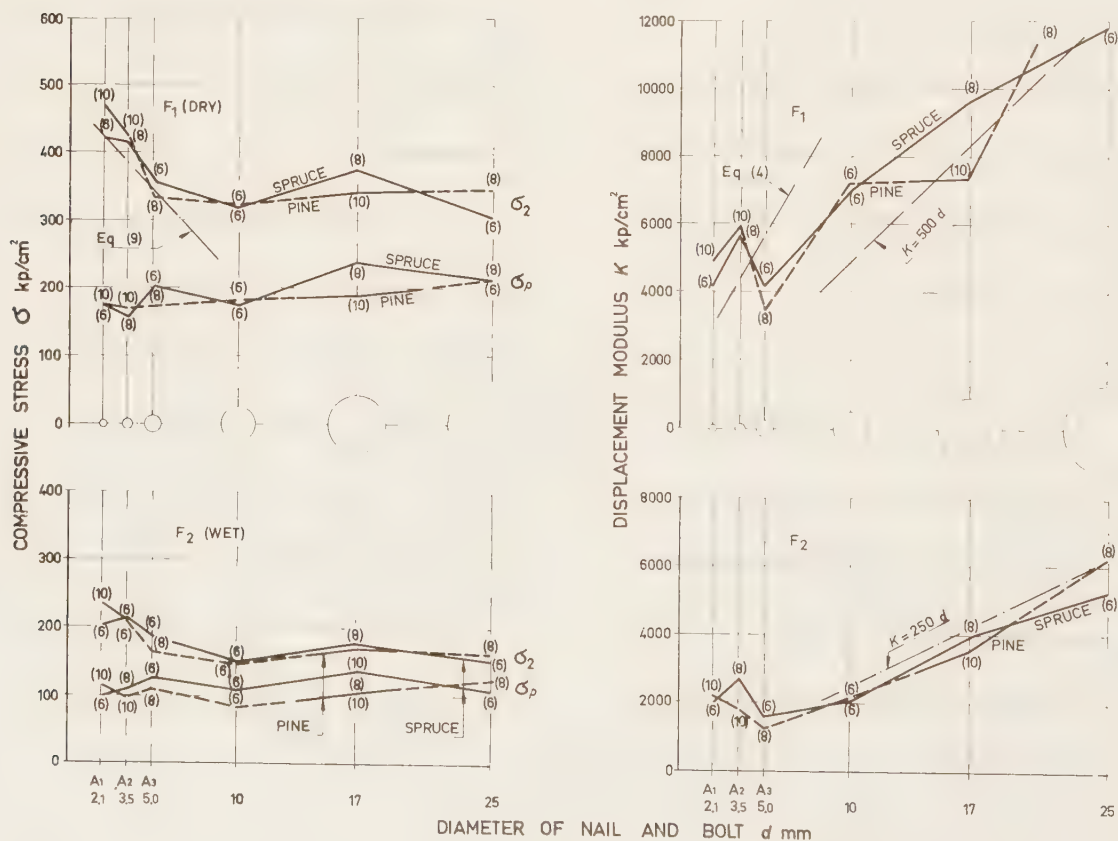


FIG. 22. Diameter dependence of the stress and displacement values (cf. fig. 20). The dash-and-dot lines in the graphs for K indicate a constant displacement modulus for nails, but for bolts a modulus that is proportional to K (that is proportional to the diameter ($d \geq 8$ mm)).

The diameter dependence is shown in fig. 22. As regards the strength under nails in prebored holes, the result indicates that the level of the strength as well as the dependence on the diameter are of the same order as the strength obtained without prebored holes. The strength under bolts, on the other hand, shows no dependence on the diameter. The proportional limit for the displacement of nails in prebored holes, approximately 50 % of the strength, rather surprisingly, is about the same as in the tests without prebored holes. For bolts the proportional limit is about 60 % of the strength. In soaked timber, the stress values obtained were about 50 % lower than in dry timber.

Even if preboring for the nails had no significant influence on the displacement above the proportional limit, it resulted in a considerably higher value of the displacement modulus (K). No conclusions should be drawn from the results as regards the diameter dependence of the K value for the nails in prebored holes (diameters 5 mm or less). On the other hand, the K value was clearly dependent on the diameter at larger diameters (bolts, dowels). Summarizing the results according to fig. 22, a reasonable value of the displacement mod-

ulus for calculating the initial displacement in prebored holes in dry pine and spruce parallel to the grain should be 4000 kp/cm² (400 N/mm²) [soaked timber 2000 kp/cm² (200 N/mm²)] if the diameter is below 8 mm. For diameters from 8 to 25 mm, the results correspond approximately to a K value that is directly proportional to the diameter: $K = 500 d$ (soaked timber $K = 250 d$) kp/cm², d in mm. This K value is lower than the value $K = 700 d$ previously suggested and based on tests on double-shear joints with bolts and dowels, see fig. 23.

In the Swedish building code, BABS 1960, the working loads were based on the assumption that the strength under the bolts in pine and spruce is 0.7 times the compressive strength of the timber parallel to the grain, as determined on prisms.¹ At a density of 0.45 g/cm³ this would, according to fig. 14, give a strength of 320 kp/cm² (32 N/mm²), i.e. a value well in conformity with the value of σ_2 according to fig. 22.

¹ In the revised edition of 1967 this strength ratio depends on the angle between the force and the fibre direction.

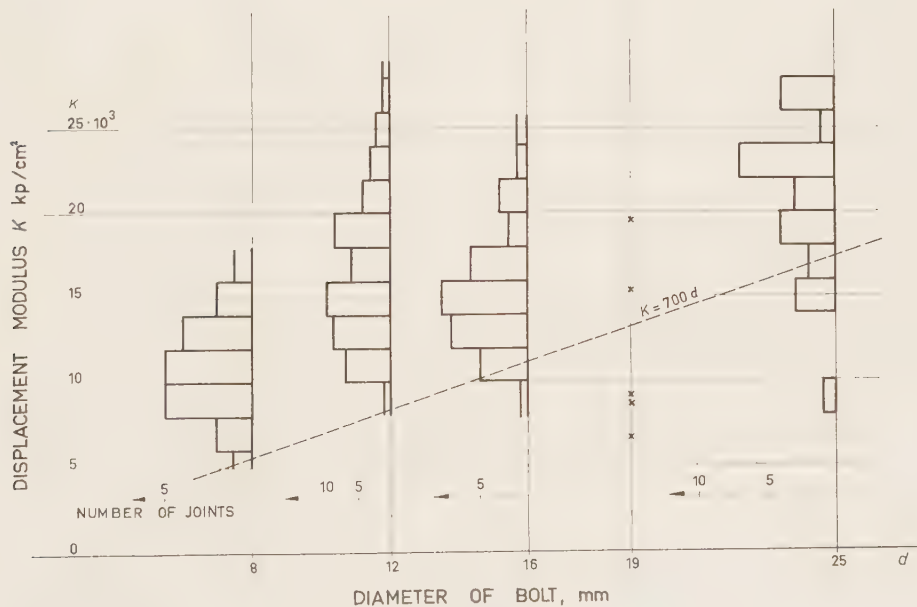


FIG. 23. The displacement modulus K according to earlier tests of pine and spruce joints with dowels and bolts (Norén, 1948).

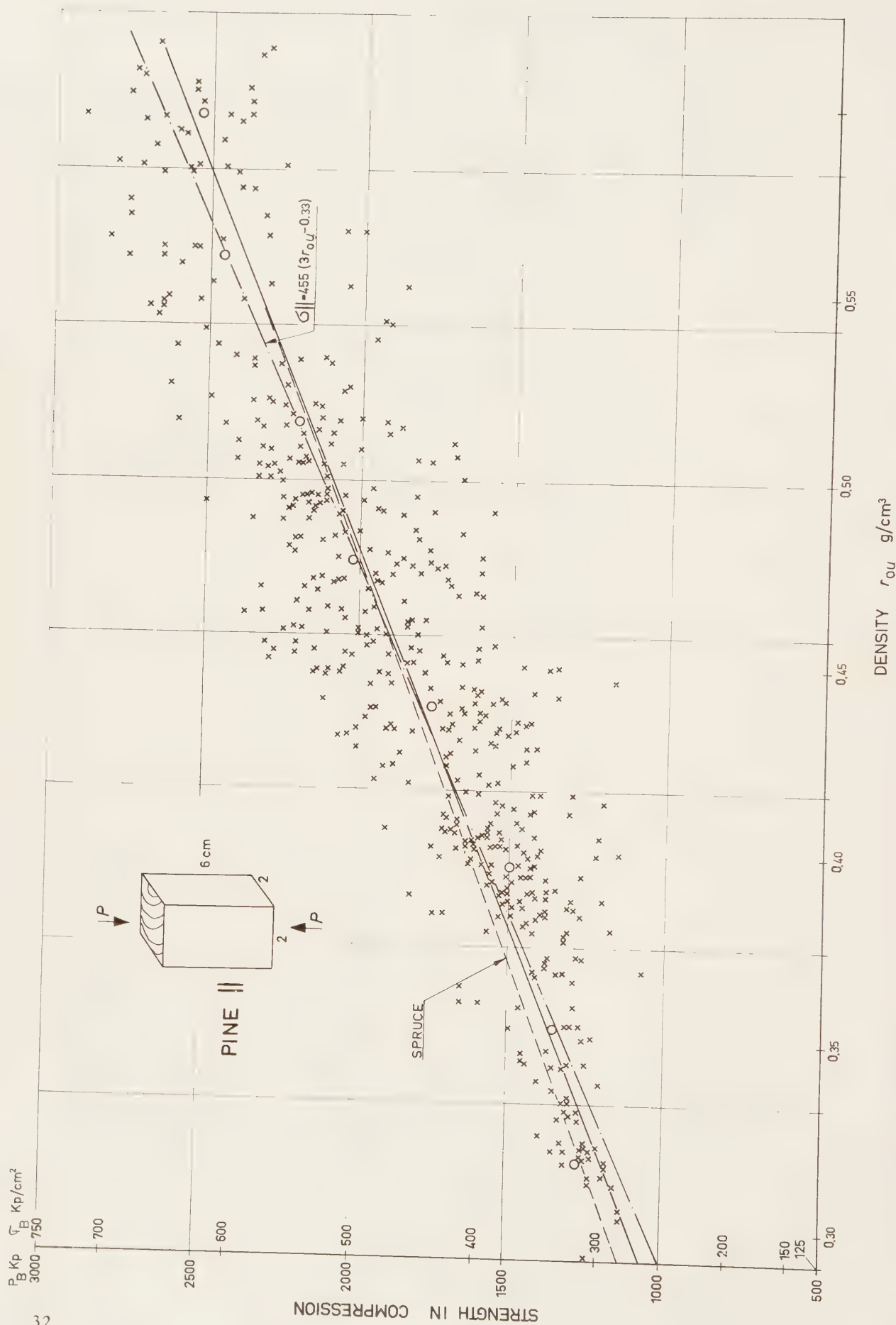


FIG. 24. Correlation between density and strength parallel to the grain in pine determined on control prisms, stage 1. The continuous line, fitted to the sub-means (circles) by means of least squares, agrees well with the density dependence found for different stresses at the displacement tests (dash-and-dot line). The regression line for spruce is reproduced from fig. 25.

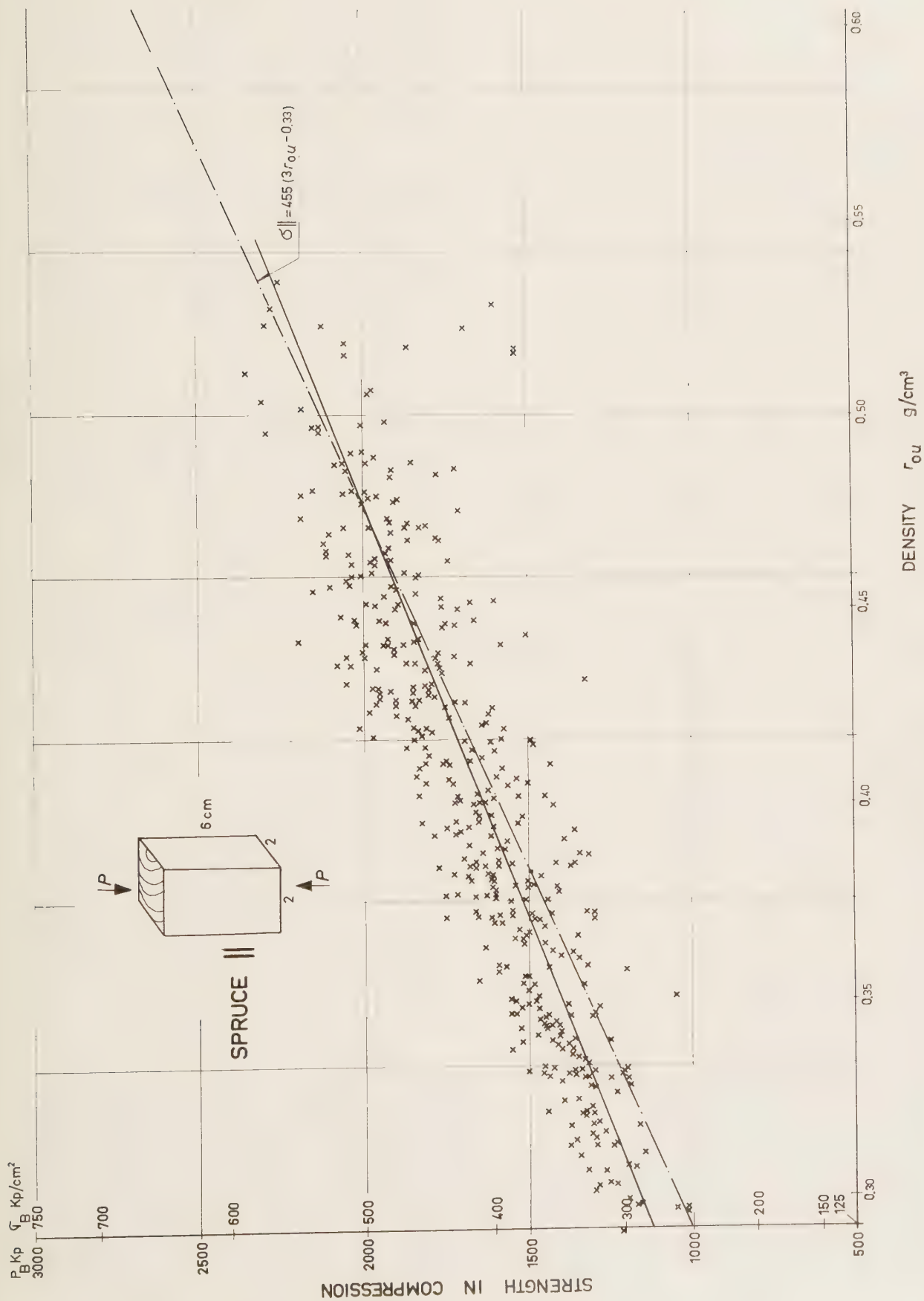


FIG. 25. Strength of control prisms of spruce, cf. fig. 24.

CREEP IN DISPLACEMENT OF NAILS

Introduction

In previous papers the author dealt theoretically with deformations in nailed joints due to loading including creep (Norén, 1961), (Norén, 1962). The influence of the design of the joint was demonstrated and it was stated that, though creep may occur at a very low load when slender nails are used, the creep is retarded by the (plastic) bending of the nails.

In this chapter the author presents and discusses results of long-term tests on nailed joints. The purpose of these tests, as well as of those described in first section, was to study the reaction of wood exposed to stress by laterally loaded nails which remain approximately straight during their movement in the wood. The design of the joints was similar to that of the joints used for the short-term tests under different conditions (density, moisture content, size and shape of the nail cross section, etc.), though, in this case, the number of variables was reduced.

Definition

The definitions and notations used in the paper referred to above (Norén, 1962) have as a rule, been used here too. The relation between the compression q and the lateral displacement y of the nail was defined by (1) and (2)

$$\sigma d = q = K(y)y \quad (13)$$

in which d is the nail diameter and $K(y)$ a function of y . The displacement y can be considered to be the sum of the components:

$$y = y_I + y_H + y_{nH} \quad (14)$$

where y_I denotes the "initial" displacement that may occur when the nail is loaded for the first time. The component may be due to unsatisfactory fitting of the nail in the nail-hole or to other irregularities. The short-term tests described in first section show that y_I is generally small and can thus be neglected.

The components y_H and y_{nH} represent the parts of the displacement respectively obeying and not obeying Hooke's law.

Apart from the differentiation of the displacement in (14), we may also divide it into an instantaneous part and a time-dependent part:

$$y = y_0 + y_t \quad (15)$$

We will now consider y_I as well as y_H a part of y_0 . The component not obeying Hooke's law y_{nH} is, however, assumed to be in part instantaneous and in part time-dependent:

$$y_{nH} = y_{nH_0} + y_{nH_t} \quad (15 \text{ a})$$

If we write the "generalized Hookean displacement"

$$y_{gH} = y_H + y_{nH_0} \quad (16)$$

we can write:

$$y_{gH} = y_0 - y_I \approx y_0 \quad (17)$$

Obviously, the components of y , so far defined, can be elastic or non-elastic (irrecoverable), irrespective of whether they are time-dependent or not. (It should be observed that the elasticity theory applies to y_H only, not to elastic displacement generally.) We may thus dissolve the time-dependent component into

$$y_{nH_t} = y_{g(NH)} + y_{gN} \quad (18)$$

The last term of (18) represents the *flow*, which, by the usual definition, is an irrecoverable time-dependent deformation. The subscript "gN" indicates "generalized Newtonian flow" of which the Newtonian or "pure" liquid flow y_N is a special case occurring when the rate of the flow is proportional to the stress. The term $y_{g(NH)}$, correspondingly, represents the generalized retarded elastic displace-

ment, a special case of which is the *visco-elastic* displacement y_{NH} , which, like y_N , is proportional to the stress. The subscript "NH" indicates that y_{NH} is equivalent to the extension of a rheological model composed of Newtonian and Hookean elements. (For elastic-viscous deformation which is only partly elastic the subscript "HN" should be preferred.)

In (Norén, 1962) the time-dependent deformation of the nails as well as of the wood was referred to as elasto-plastic. In common rheological nomenclature "plastic" and "plasticity" are attributed to materials which flow above a limit of stress below which they behave elastically. This definition is perfectly applicable to the deformation of nails, whereas the existence of a definite flow limit for wood, especially in deformation under nails is unlikely and has not been proved.

Creep y_c is here defined as time-dependent displacement (deformation) at constant load.

Theory

In order to demonstrate the influence of the design on the creep in nailed joints the author (Norén, 1962) previously assumed a linear relation between y and q

$$y = y_H + y_{NH} = \frac{q}{K_1} + \frac{q}{K_2} \left(1 - e^{-t/\tau}\right) \quad (19)$$

A possible generalization is

$$y = q \sum_{K_i} \frac{1}{K_i} [1 - \exp(-t/\tau_i)] [1 - (q_R - q) \beta_i] \quad (20)$$

$(i = 1, 2, 3, \dots, n)$

This may be written in a more concentrated form:

$$y = q \sum_{K_i} \frac{1}{K_i} \Psi_i \Phi_i \quad (21)$$

where Ψ_i is a time function and Φ_i a stress function. Ψ_i/K_i represents the deformation per unit load of a Kelvin (Voigt) body at the time t after the application of the load as a fraction of the total deformation after infinite time (the "creep amplitude"):

$$y_{i\infty} = q/K_i \quad (22)$$

The stress function, in which q_R is a reference load and β_i a "coefficient of non-linearity", is a simple correction for the non-linearity of the deformation.

The number of terms (value of n) in (20) depends on the time-range of the experiment as well as on the accuracy aimed at in describing the actual results. The limited speed at which the joints is generally loaded is one good reason for introducing an *instantaneous* displacement. By assuming $\tau_1 \ll t$ we define it as the first term ($i = 1$) of equation (20):

$$y_0 = y_1 = \frac{q}{K_1} [1 - (q_R - q) \beta_1] \quad (23)$$

With reference to (16) and (17) we may write instead of (23)

$$y_0 = \frac{q}{K_H} + \frac{q^2 \beta_1}{K_1} \quad (23 \text{ a})$$

in which the Hookean displacement modulus ("slip modulus") K_H represents the slope of the "extreme" short-term $q-y_0$ -curve at the origin:

$$K_H = \frac{K_1}{1 - q_R \beta_1} = \left(\frac{dq}{dy} \right)_{q=0} \quad (24)$$

If β_1 in (23 a) is substituted we get

$$y_0 = \Phi_1 q/K_H \quad (25)$$

in which Φ_1 is the stress function:

$$\Phi_1 = 1 + \frac{q}{q_R} \left(\frac{K_H}{K_1} - 1 \right) \quad (26)$$

By introducing *the creep amplitude*

$$a_i = \frac{q}{y_0 K_i} \quad (27)$$

we may write *the relative creep* (specific creep):

$$y/y_0 - 1 = \sum_2^n a_i \Phi_i \Psi_i \quad (28)$$

From this *generalized* creep function different special cases may be deduced, e.g.:

A. $\Phi_i = \Phi_1$ gives a relative creep which is *independent of the stress* (q):

$$(y/y_0 - 1) = \sum \frac{K_1}{K_i} \Psi_i \quad (29)$$

A sub-case is obtained when $\Phi_i = 1$. This corresponds to the linear deformation of Kelvin bodies in series.

B. Another special case is obtained when the relative creep is *proportional to the stress* (either q or σ). This is obtained by inserting in (28):

$$\Phi_i = y_0/y_{R0} = \frac{q}{q_R} \Phi_1 \frac{K_1}{K_H} \quad (30)$$

which gives us the relative creep:

$$(y/y_0 - 1) = \frac{q}{q_R} \sum \frac{K_1}{K_i} \Psi_i \quad (31)$$

C. A third special case, which makes the creep still more influenced by the stress, is introduced by the stipulation

$$\Phi_i = \frac{q_R}{q} \left(\frac{y_0}{y_{R0}} \right)^2 \quad (32)$$

The corresponding relative creep is *proportional to the instantaneous deformation* y_0 :

$$(y/y_0 - 1) = (y_0/y_{R0}) \sum \frac{K_1}{K_i} \Psi_i \quad (33)$$

From (23) we have:

$$y_{R0} = q_R/K_1$$

The formulas given should be regarded as examples of phenomenological expressions of non-linear creep. There is no obvious physical reason why Φ_i should have any of the values used here.

Methods and Scope of Tests

For the investigation of creep dealt with in this paper the types of joints were selected among those used for previous short-term tests. The nails were of a type manufactured and preferred for most structural nailing (Swedish Standard SMS 1382). The cross section is a modified square. The tensile strength is about 10^5 lbs. per sq. in. Here this is immaterial, however, since no bending of the nails was allowed to occur.

The wood used was pine (*Pinus Silvestris*). The density was either of the order 0.35 to 0.40 g/cm³

or 0.45 to 0.50 g/cm³, whereas the moisture content was $(10 \pm 1) \%$ in all specimens.

The joints were of the symmetric double-shear type, the main member where the displacement took place being of wood and the side members of steel. There was only one nail in each joint. Generally, six joints (three wood members) were connected in a series to a chain and three such chains were as a rule tested at the same time, see fig. 26. The only difference was the level of loading. Ten groups of chains were tested (some of them with a reduced number of joints), in certain cases in repeated test series. At the tests the fibre direction of the wood was either parallel ($\parallel$) or perpendicular ($\perp$) to the direction of the load. The diameter (thickness) of the nails was either 3.2 or 5.0 mm and the corresponding thickness of the wood members 13 and 20 mm. Thus, a load of 100 kp (1000 N) applied to a chain of joints with 5.0 mm nails gave a compression stress under the nails of 100 kp/cm² (10 N/mm²). This case is denoted as 50 $\parallel$ 100 or 50 $\perp$ 100. Two main types of loading were used. The first type was a constant load of long duration. In this case the loading intervals were about three months and the intervals when the joints were allowed to recover one to three months. The second type was repeated loading and recovery at intervals of one week.

The displacement was measured with dials as slip in the joints generally. (In measuring the creep immediately after loading, the dials were photographed in certain series. In a few series clips with electrical strain-gauges were used instead of dials.)

Instantaneous deformation

The instantaneous deformation observed in the nailed joints when loaded for creep studies, makes it possible to draw extremely short-term load-slip curves. Each curve in fig. 27 represents the deformation of three chains of six joints at instantaneous application of the load from zero stress to the different stress levels at which the creep was to be studied. The test specimens were selected with the aim of making the three chains in a series as equal as possible.

The circled values are mean values of the displacement in six joints 15 s after the full load was applied, the application itself having taken about 5 s. The deformation reproduced is thus not truly

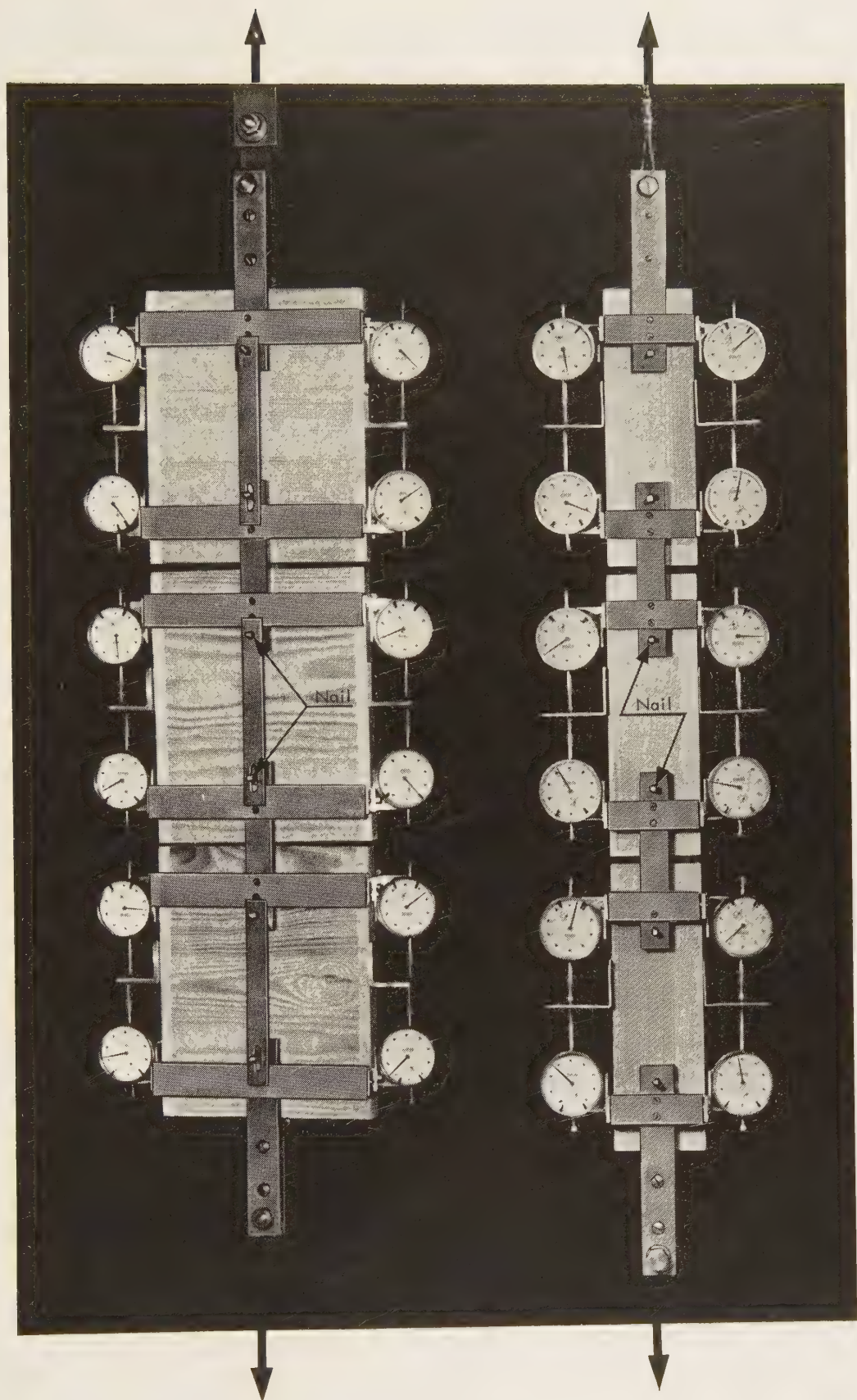


FIG. 26. Chains of joints under sustained load perpendicular and parallel to the grain.

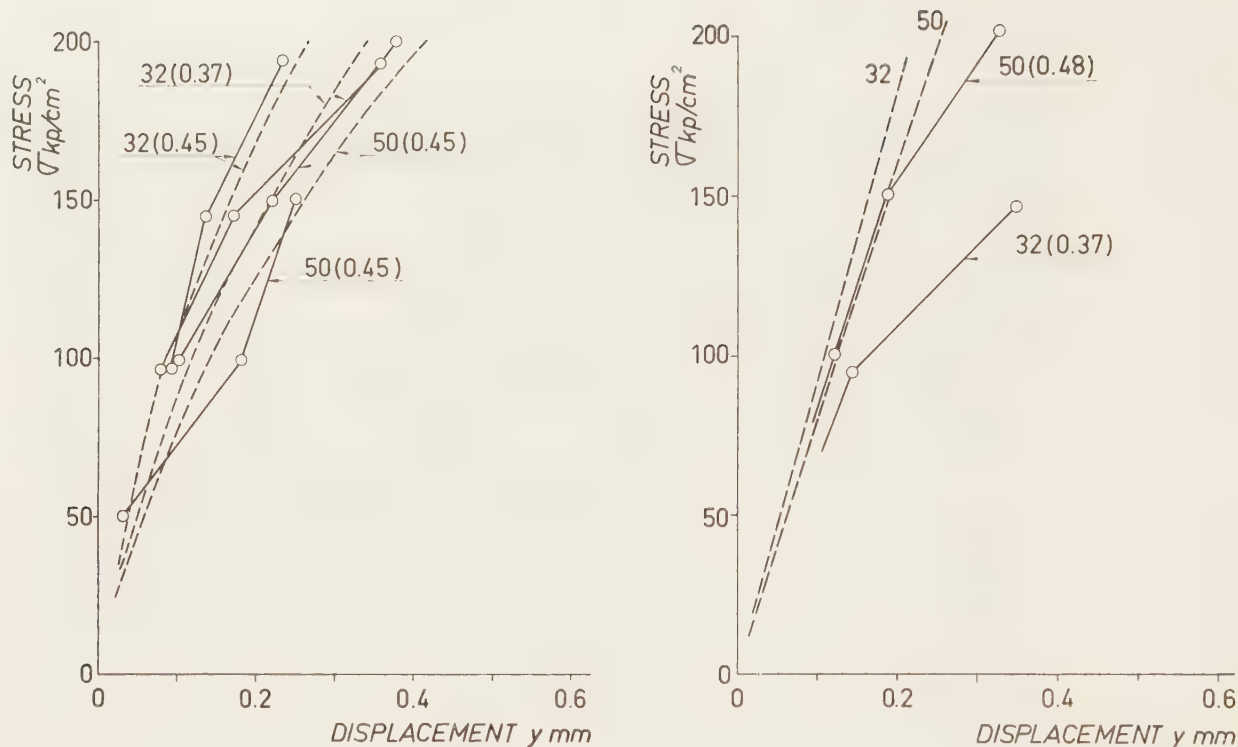


FIG. 27. Instantaneous displacement of nails No. 32 and No. 50. Each circle represents the mean value for six joints, i. e. one chain of joints loaded directly from zero stress to the indicated stress level. In brackets the density (g/cm³) of the wood (*Pinus Silvestris*). The broken curves show the calculated displacement. To the left loading parallel and to the right perpendicular to the grain.

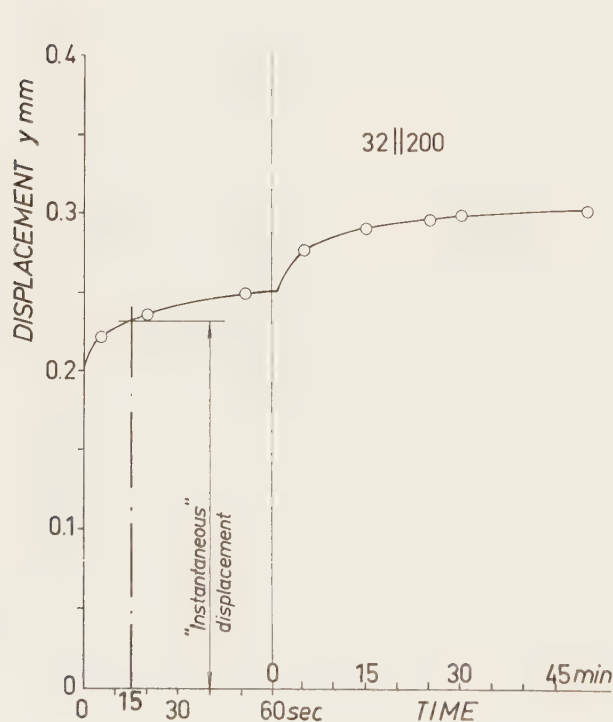


FIG. 28. The method of loading made it difficult to measure the creep during the very first seconds accurately. The displacement 15 sec. after the application of the load was therefore regarded as "Instantaneous".

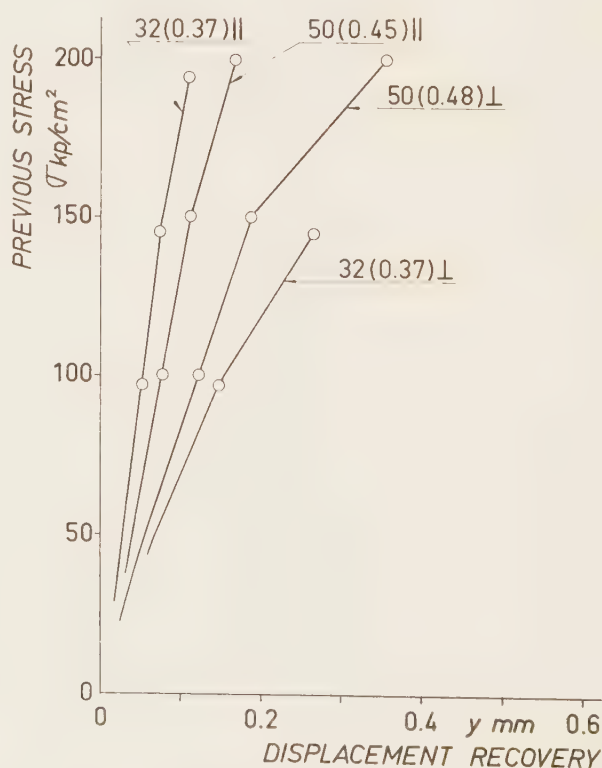


FIG. 29. Instantaneous recovery at the removal of the load, i. e. when the stress was changed directly from the indicated level to zero. The joints (cf. fig. 27) had been loaded for approximately three months.

instantaneous (y_0), but includes creep. This creep may, as seen in fig. 28, reach a magnitude of $0.1 y_0$. Considering the general deviation of the test results and the fact that the reliability of the load value and the measured deformations during the very first seconds may be questioned, it was found reasonable to adopt the 15 s value of the deformation as "instantaneous". This approximation does not change the main conclusion that Hooke's law is not applicable, except with considerable error, when the stress exceeds 100 kp/cm^2 (10 N/mm^2) or when y_0 exceeds 0.1 mm ; even if the duration of loading is extremely short.

The broken-line curves in fig. 27 are calculated from (24)

$$y_0 = \frac{\sigma d}{3r_{0u} - 0.33} \left(\frac{1}{6000} + \frac{\sigma}{80 \times 10^4} \right) \quad (34)$$

y_0 and the nail diameter d are here expressed in cm, the stress σ in kp/cm^2 , and the density of the wood, r_{0u} , in g/cm^3 . The correction factor $(3r_{0u} - 0.33)$ for the deviation of the density from the reference value 0.445 g/cm^3 , is adopted from the results reported in first section. Also the value $K_H = 6000 \text{ kp/cm}^2$ (600 N/mm^2) was adopted on the basis of these previous tests, whereas the value

$$\frac{K_1}{\beta_1 d} = \frac{\sigma_R}{1/K_1 - 1/K_H} = 80 \times 10^4 \text{ (kp/cm}^2)^2 \quad (34 \text{ a})$$

was chosen as approximately coincident with the experimental results according to fig. 27. [In order to obtain a better agreement with the test results, a term of a higher order must be used in (23 a).]

Fig. 27 also shows the displacement at extremely short-term loading perpendicular to the grain. Though, in this case, there was no loading at levels below 100 kp/cm^2 (10 N/mm^2) the initial slope of the curves can be estimated to correspond approximately to the K_H value obtained at the short-term tests with stepwise loading: $K_H \approx 3500 (3r_{0u} - 0.33) \text{ kp/cm}^2$. The joints thus show less initial stiffness perpendicular to the grain than parallel to it. The y_0 values of the 50-nails were about the same in both directions. On the other hand, the displacement of the 32-nails was very large when perpendicular to the grain even at higher stress levels. This is remarkable since the "flow value" of stress σ_2 (value giving 2 mm displacement), according to figs. 11c,d, should increase with decreasing nail diameter.

The curves in fig. 29 show the instantaneous recovery when the load was removed after three months. The chains of joints and the method of deducing the curves are the same as in fig. 27. When the direction of the load is *perpendicular* to the grain, the curves of recovery are very similar to the loading curves. This indicates that *the instantaneous displacement is instantaneously elastic*. The recovery after loading parallel to the grain is approximately linear. Furthermore, the slope of the lines coincides with the initial slope (tangent at the origin) of the y_0 curves at loading, see fig. 27). This indicates that *the Hookean part of the immediate displacement (y_H) is instantaneously elastic*, whereas the nonlinear part (y_{nH}) does not recover at all, cf. fig. 30. As will be shown later a similar influence of the loading direction was noticed for creep.

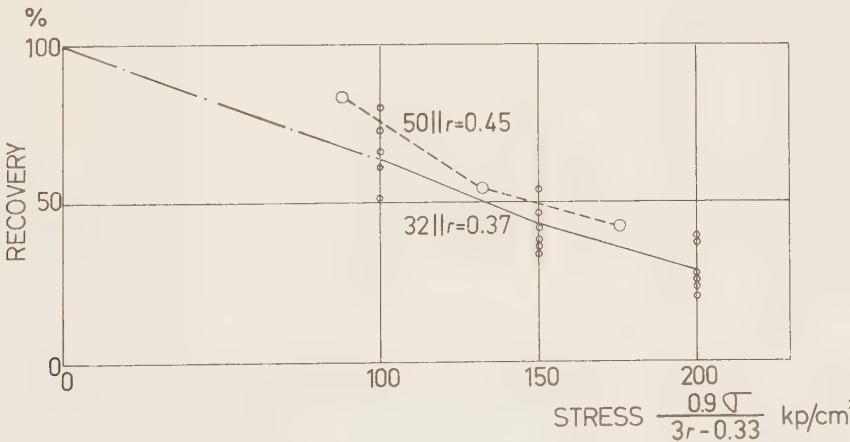


FIG. 30. The instantaneous recovery as a percentage of the instantaneous displacement (at loading) decreases rapidly with increasing stress when the load is parallel to the grain. (r = density, g/cm^3 .)

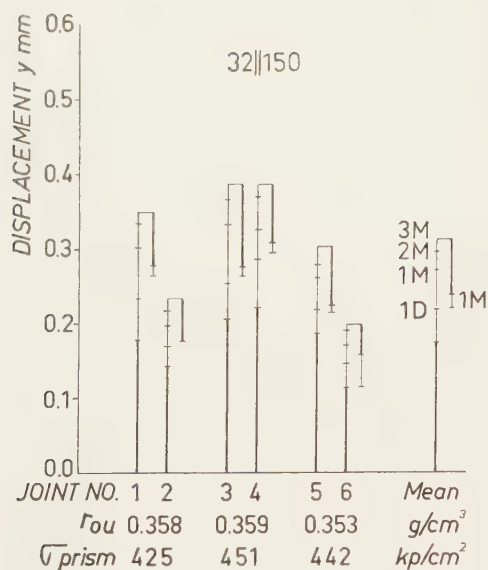


FIG. 31. Deviation of displacement within one chain of six joints. Thick lines = immediate displacement, thin lines = creep, D = day, M = month, r_{ou} = density, σ_{prism} = compression strength of control prisms.

Since the figures 27 and 29 show mean values, it may be necessary to say something about the deviation of the results within the chains. In fig. 31 the displacement of each nail as well as the mean value at one specific test are shown schematically. Though the deviations in density (r_{ou}) and strength of the wood (σ_{prism} = compressive strength parallel to the fibres of prismatic control specimens) are small (standard deviation 4 % and 7 % respectively), the deviation of the immediate displacement of the nail as well as of the creep is considerable. From the deviations shown in table 6 and the fact that the standard deviation of the density within the chains was about 4 %, it follows that a standard deviation of the order of about 25 % should be expected in testing single nails when the wood is properly matched. We may consequently expect a difference between the highest and the lowest value that will—with 2.5 % probability—exceed the mean value. This large deviation, which is due to the rather irregular embedding conditions associated with common nailing, is a considerable disadvantage in testing single-nail joints. It should also be pointed out that, owing to the non-linearity of the displacement, the mean value for the six joints in series will slightly exaggerate the displacement of one joint with six nails.

Chains of joints were also loaded intermittently. The load was applied during one week and then removed for one week. This procedure was repeated two or three times. Joints with a "loading history" were also subjected to such cyclic loading. These

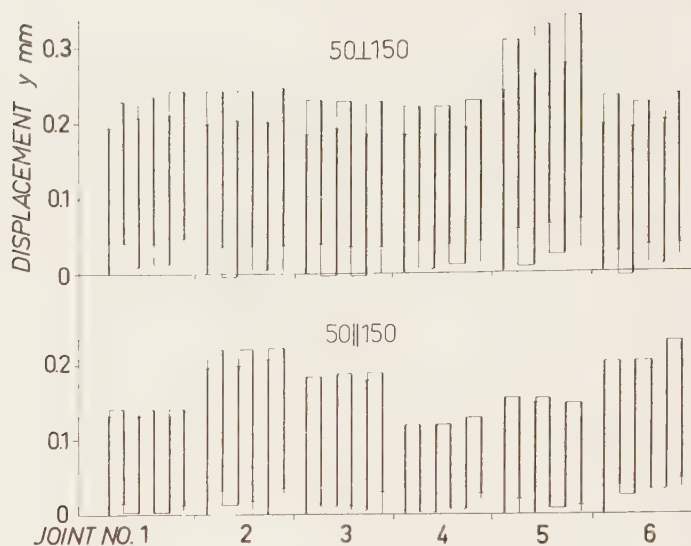


FIG. 32. Displacement at intermittent loading in joints which had previously been exposed to long-term loading. Cf. fig. 33.

joints had previously been exposed to constant loading for three months and had then been allowed to recover for one month. As exemplified in fig. 32, the previously loaded joints behaved almost perfectly elastically—except for the creep parallel to the grain.

The joints without a loading history behaved differently (see fig. 33). In the first place, there is a

TABLE 6. Standard deviation of nail displacement (y) within the chains of six joints as a percentage of mean displacement.

1. Immediate displacement (y_0) after loading.
2. Creep (loading time 1 to 3 months).
3. Immediate recovery after removing the load.

Type of joint	Stress kp/cm^2	y_0 mm	Standard deviation, %		
			1	2	3
32	100	0.085	25	23	23
	150	0.17	23	34	30
	200	0.39	25	16 ^a	10
50	100		17	—	20
	150	0.24	26	35	15
32	100	0.088	22		
	150	0.136	36		
	200	0.23	26	—	
50	100	0.09	22	37	18
	150	0.224	40	34	30
	200	0.436	48	40	16
Mean value			39		
32 ⊥	100	0.160	30	56	24
	150	0.348	34	58	36
50 ⊥	100	0.124	9	6	9
	150	0.197	8	17	5
	200	0.337	12	48	21
Mean value			19	37	19

^a One extremely high value excluded.

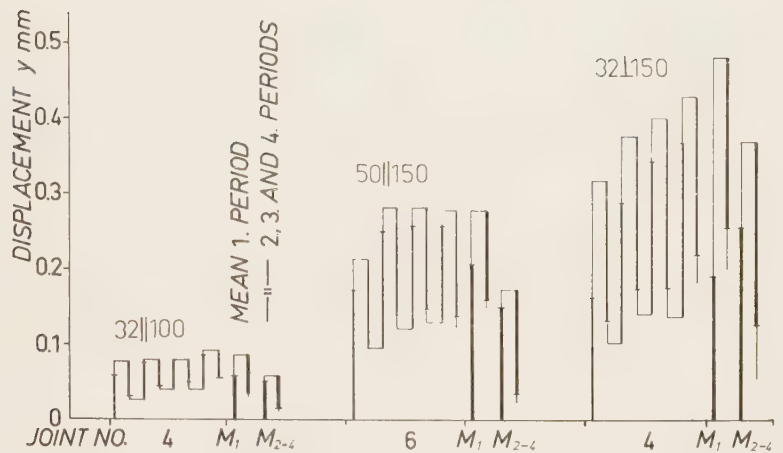


FIG. 33. Displacement at intermittent loading in joints without a loading history. Thick lines = immediate displacement, thin lines = creep.

spectacularly large displacement at the first loading, a large part of which does not recover. In the second place, the displacement increases with the number of loading cycles. This is to a certain extent due to the fact that the instantaneous displacement at loading was slightly larger than the instantaneous recovery when the load was removed. However, the principal reason is that the recovery creep is smaller than the creep during loading intervals. (It should be noticed that the mere fact that this difference in the creep was observed does not permit of any conclusions as to the elasticity of creep). Perpendicular to the grain, the immediate recovery slightly *exceeded* the immediate displacement recorded at the two or three first loadings.

Generally, the instantaneous displacement in the joints at intermittent loading, parallel as well as perpendicular to the grain, attained a constant value after a few periods. This value was almost the same when the load was applied as when it was removed. Table 7 gives the mean values of K_0 calculated on the instantaneous recovery after long-term creep and after short-term creep (mean values from intermittent loading). The difference between the respective values is very small. As an overall mean value we may adopt $K_0 \parallel = 6000$ ($3r_{0H} - 0.33$) kp/cm^2 when the load is parallel to the grain, i.e. $K_0 \parallel = K_H$. Perpendicular to the grain we may as a first approximation adopt $K_0 \perp \approx 0.5 K_0 \parallel$, though the results indicate that, in this case, the modulus depends on the stress and, possibly, on the diameter of the nail as well.

TABLE 7. Instantaneous displacement recovery modulus K'_0 after long-term creep (LC) and at intermittent loading. (INT Period = 14 days. Specific gravity = r_{0H} . Stress = σ .)

Type of joint	No. of joints	Type of test ^a	r_{0H} g/cm ³	σ kp/cm ²	K'_0 ^c kp/cm ²
32	6	LC	0.357	100	5700
	4	INT4	0.394	100	6140
	6	LC	0.357	150	6000
	4	INT4	0.396	150	5630
	6	LC	0.362 ^b	200	5450
	6	(INT3)		200	5350
50	6	LC	0.434	50	8950
	6	INT4	0.452	50	5430
	6	LC	0.464	100	3970
	6	LC	0.450	100	6560
	6	INT4	0.441	100	7600
	6	LC	0.461 ^b	150	6120
	6	(INT3)		150	5860
	6	INT3		150	5050
	6	LC	0.450	150	6450
	6	INT4	0.454	150	5760
	6	LC	0.452	200	5960
50	6	LC	0.473 ^b	100	4030
	6	(INT3)		100	3980
	6	LC	0.489 ^b	150	3860
	6	(INT3)		150	3770
	6	LC	0.471	200	2900
32	6	LC	0.366	100	2380
	6	INT4	0.371	100	2400
	6	LC	0.376	150	2060
	6	INT4	0.384	150	1930

^a Long-term creep test.

INT4 Intermittent loading: 4 periods, each consisting of 7 days loading and 7 days recovery.

^b (INT) Periods longer than 14 days. Repeated testing of the same joints.

^c $K_0 \approx K'_0 (3r_{0H} - 0.33)$.

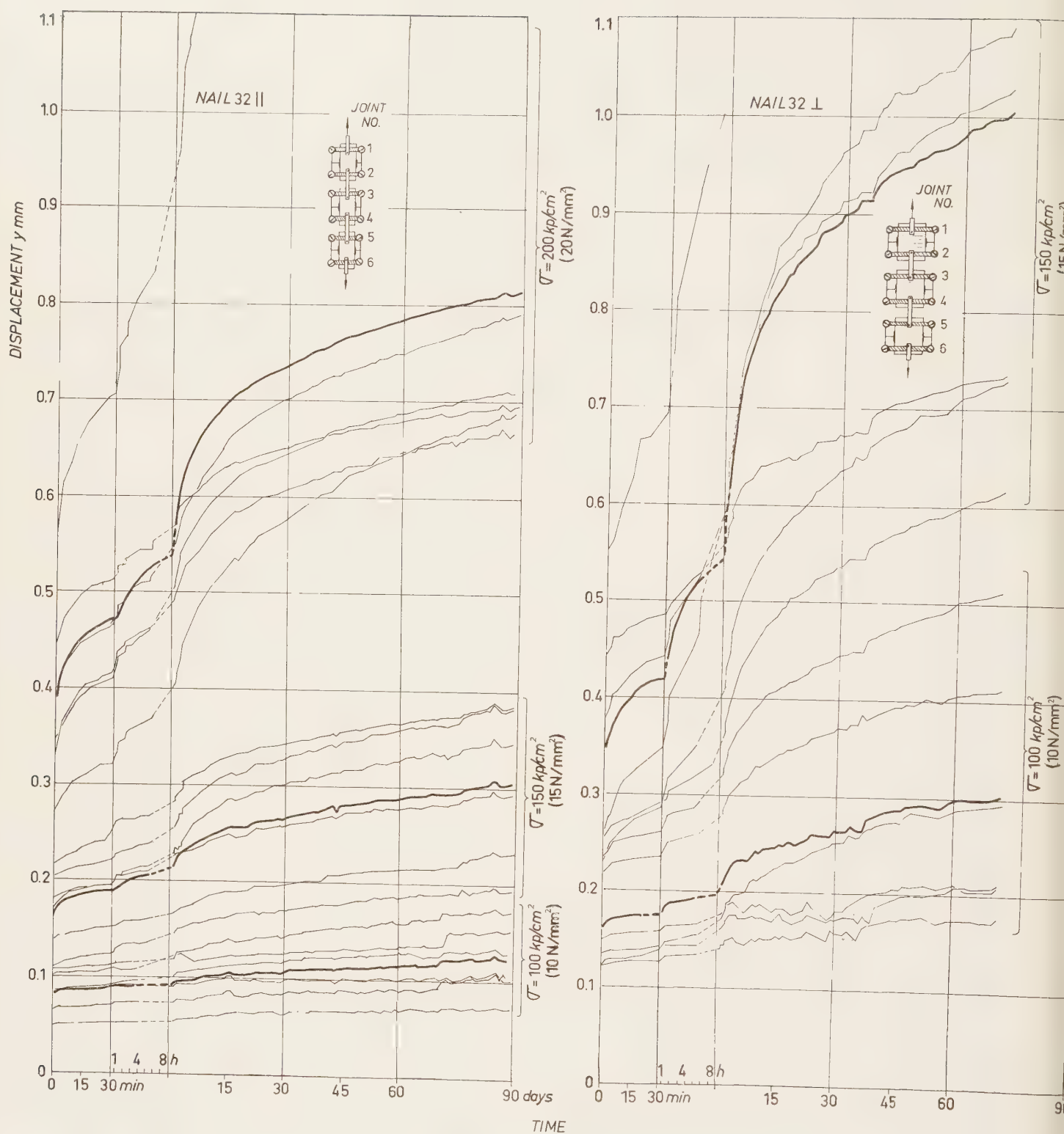


FIG. 34. Creep in joints with No. 32 nails (3.2 mm diameter) at the first loading. The thick curves show the arithmetic mean of the displacement in the six joints constituting a chain. Similar curves were obtained with No. 50 nails. (Note the different time scales.)

Creep at First Loading

We have shown that the displacement parallel to the grain, when the nail is loaded for the first time, includes a large portion of "permanent set". Since this set increases with stress and time, it is not identical with, but includes the "initial" displacement y_1 in (14). Anyway, when giving the creep of a nailed joint, we have also to specify if it is the first loading (the level of stress in question) or if the joint has a loading history.

Creep curves for some of the joints after the first loading are shown in fig. 34. For the sake of simplification, only the mean curves for the six joints in the chains will be considered in the following. As the creep deviates considerably within the chains,

two things should be observed. In the first place, *the level* of the mean curve may not be representative of the "population", but this is of minor importance as far as *the relative creep* is concerned. In the second place, when the creep is increasing more than proportionally to the immediate displacement, the arithmetic mean of the creep will be slightly larger than the creep corresponding to the mean immediate displacement, i.e. the relative creep will be slightly exaggerated. However, the similarity of the mean curves and the adjacent individual curves in fig. 34 shows that the error is small.

The relative creep in the joints when loaded for the first time is reproduced in fig. 35. It is clearly shown that the relative creep is not constant but

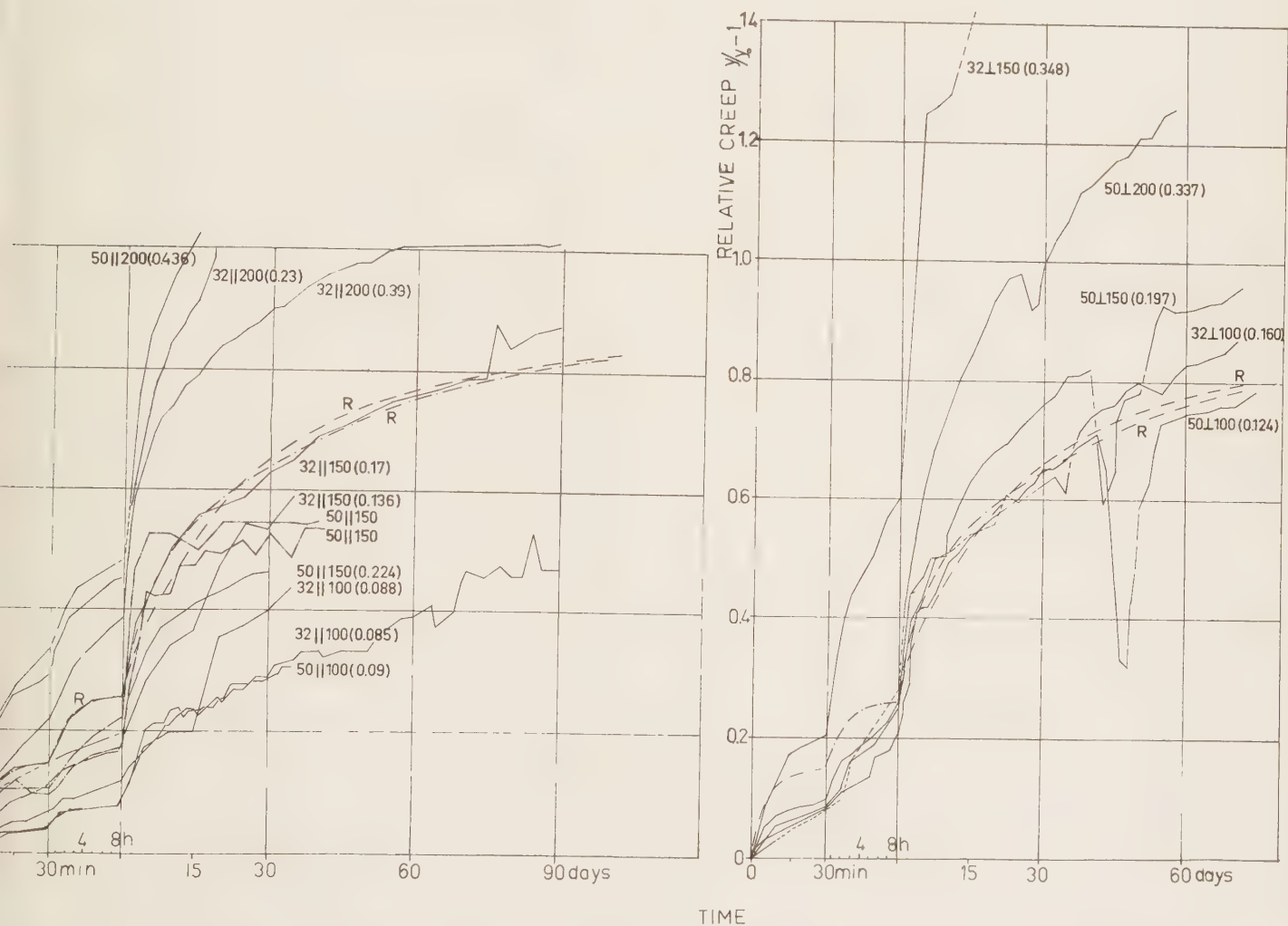


FIG. 35. Relative creep, i.e. the ratio of the creep to the instantaneous displacement, y_0 . (The values in brackets give y_0 in mm.) The curves show the mean relative creep in the chains. The dash-and-dot and broken curves represent the reference functions R and R' respectively. || = parallel to the grain (to the left), ⊥ = perpendicular to the grain (to the right).

increases with increasing stress (immediate displacement). Though some highly stressed joints ($y_0 \geq 0.35$ mm) show spectacularly large deviations, which seem to be due to the nail diameter, the curves do not permit any reliable conclusion to be drawn as to the possible influence of an interaction between the nail diameter and the immediate displacement on the creep.

The curve of reference (R) is the same in both diagrams, representing loading parallel ($\parallel$) and perpendicular ($\perp$) the grain, respectively. It will thus be seen that the creep in the day-scale is of the same order in both loading directions, if the immediate displacements is the same. The creep retardation, perpendicular to the grain, however, appears to be somewhat slower than parallel to the grain. The short-term creep rate (minute and hour-scale) is lower than the rate of creep after two months, as is shown in fig. 35. The reference curve (R) is a curve closely fitted to the results obtained from a particular chain of joints, loaded parallel to the grain, with a stress of 150 kp/cm² (15 N/mm²) (32 $\parallel$ 150) giving $y_0 = 0.17$ mm. The fitting is based on the assumption that the derivate $D = d(y/y_0 - 1)/d(\log t)$ represents the distribution of the creep amplitudes a_i in (14). The derivate D is plotted against $\log \tau$ and the area enclosed by the D-curve and the time-axis is differentiated into a_i values corresponding to $\tau_i = 10^{i-5}$ days ($i = 2, 3, 4, \dots, 8$). The result is the reference function (R) defined by

$$(y/y_0 - 1)_R = \sum_{i=2}^n a_i (1 - e^{-t/\tau_i}) \quad (35)$$

in which $a_2 = a_3 = a_4 = 0.085$, $a_5 = 0.12$, $a_6 = 0.265$, $a_7 = 0.14$ and $a_8 = 0.7$.

The term corresponding to $i = 1$ degenerates to instantaneous displacement. The creep term corresponding to $i = 2$ is also close to the time-frame of our experiment, $1 - 1/e = 63\%$ of the creep component a_2 being obtained already $\tau_2 = 0.001$ days ($= 1.44$ min) after the load is applied. Correspondingly, the last creep term $(y/y_0 - 1)_{R8} = 0.7 (1 - e^{-t/1000})$ is at the upper time-limit of the experiment and can below that limit ($t \approx 4$ months) be approximated: $(y/y_0 - 1)_{R8} \approx 0.7 \times 10^{-3} t$ i. e. the retardation of this creep component is so slow that it can be dealt with as flow.

The reference function can of course be simplified if the time-scale of interest is less extended than the

time-scale of the experiment described. Let us, for example, say that the creep during the first 12 hours ($t = 0.5 = t' + 0.5$) is considered as "instantaneous". The relative displacement $(y/y_0)_R$ obtained above can then be replaced by the function

$$(y/y_{0.5})_R = 1 + 0.385 (1 - e^{-t'/24}) + 0.055 \times 10^{-2} t' \quad (36)$$

This is the deformation of the much used Burger body (Maxwell and Kelvin bodies in series), fig. 36. That the functions R and R' do not deviate much in the day-scale, is shown in fig. 35.

Also, within the time limits $t = 10$ and $t = 100$ days the R-function can approximately be replaced by a straight line in a $\log y/\log t$ — diagramme, corresponding to a creep proportional to t^n .

The R function can be adopted for displacement parallel as well as perpendicular to the grain, but it is valid only for $y_0 \approx 0.17$ mm. In order to generalize it, we must define the value of Φ in (28). This is not easy to do in view of the results shown in fig. 34. Nevertheless, these results, as well as the non-linearity checking by means of the individual curves, fig. 37, indicate that proportionality between the relative creep and the instantaneous displacement may be adopted as an approximation. Consequently, the relative creep after the first loading will be, from equation (33):

$$y/y_0 - 1 = (y_0/y_{R0}) (y/y_0 - 1)_R \quad (37)$$

This assumption is followed up in fig. 38 where the relative creep is plotted in terms of y_0 . The systematic deviation, at least, has disappeared.

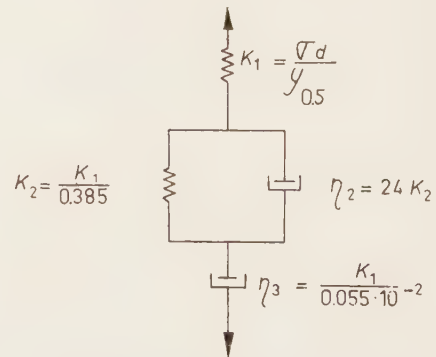


FIG. 36. Mechanical model corresponding to the reference function R'.

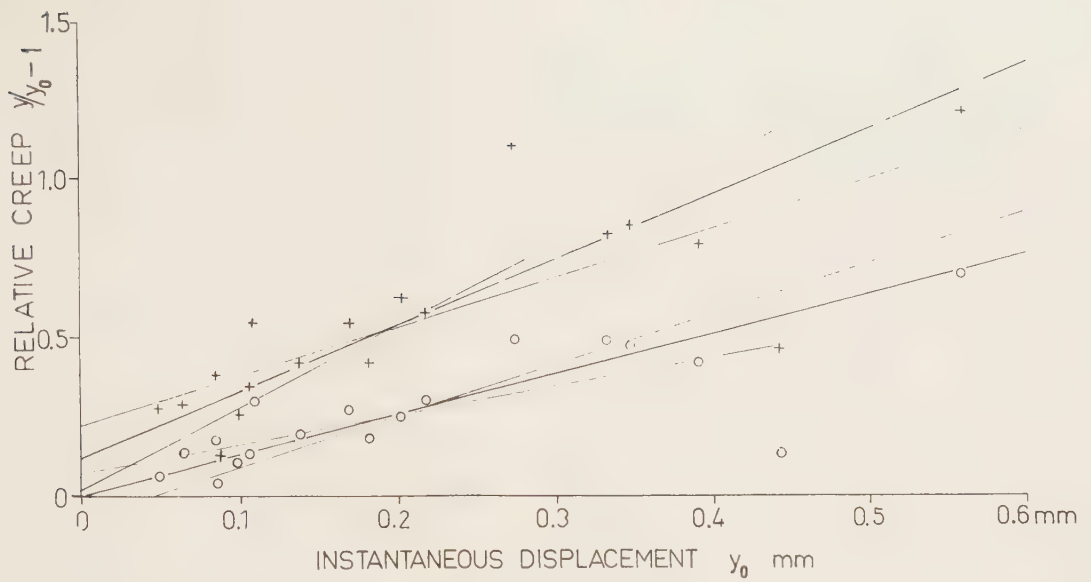


FIG. 37. Relative creep after 24 h (o) and after 1 month (+) as a function of the instantaneous displacement. Nail 32 ||, first loading.

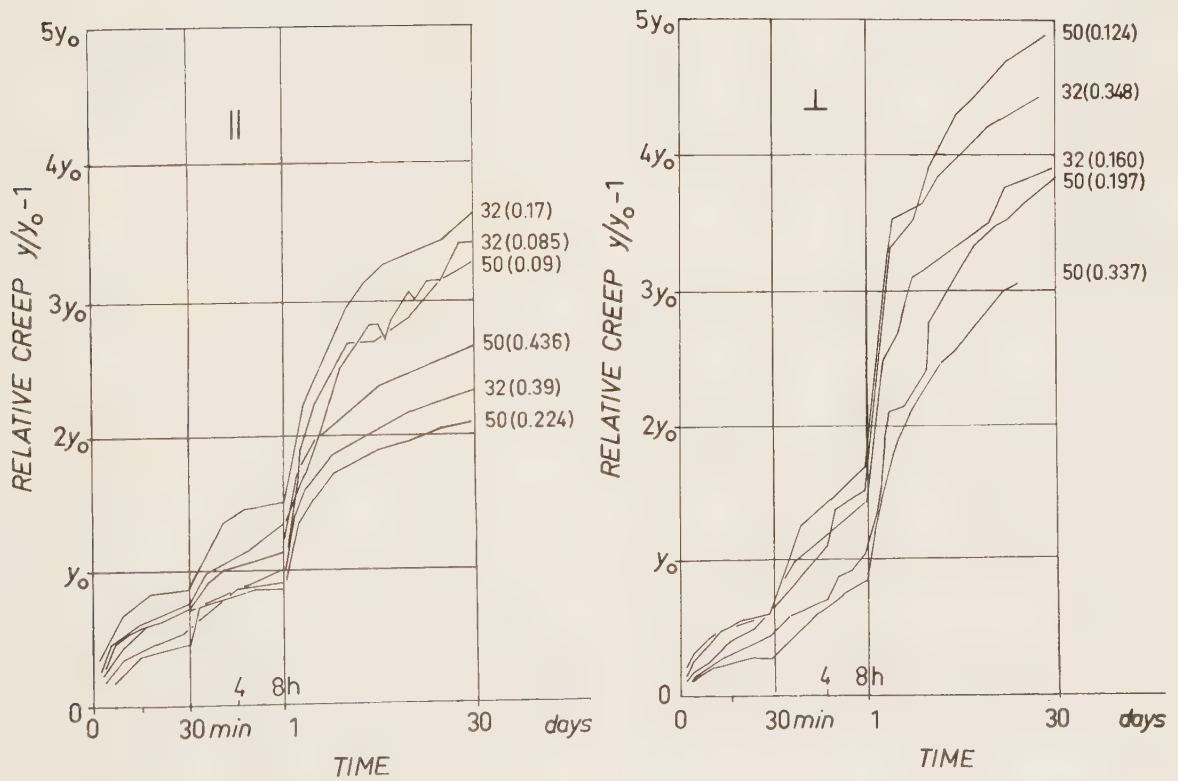


FIG. 38. Relative creep curves. Cf. fig. 35.

Recovery Creep

Once an equation and a corresponding analogy have been developed to describe the joints at their first loading, the next step is to check whether they can be applied more generally, particularly to recovery, fig. 39. Though there are several kinds of irreversibility (permanent set, strain hardening, flow, etc.) we may deal with all of them by dividing (21) into two parts:

$$y = y_\alpha + y_{1-\alpha} = q \sum \frac{\alpha_i}{K_i} \Psi_i \Phi_i + q \sum \frac{1-\alpha_i}{K_i} \Psi_i \Phi_i \quad (38)$$

and by considering y_α as reversible and $y_{1-\alpha}$ as irreversible. In the analogy of (generalized) Kelvin elements in series each element is accordingly replaced by two elements with the same retardation time τ_i but the ratio of amplitudes $\alpha_i/(1-\alpha_i)$, one element being elastic and the other able to move in the positive direction only.

If $i = 1$ is regarded as instantaneous displacement, we may estimate the value of α_1 by comparing the curves in fig. 29 with those in fig. 27. We have already concluded that, parallel to the grain, only the Hookean displacement recovers, i.e. $\alpha_1 = y_H/y_0$. The influence of the stress on α_1 is then calculated from (34). By inserting $d = 0.32$ cm, $r_{0u} = 0.445$ g/cm³ and $\sigma = 150$ kp/cm² (15 N/mm²) we get:

$$y_{R0} = 0.008 + 0.009 = 0.017 \text{ cm}$$

$$\text{and } (\alpha_{R1})_{||} = 0.008/0.017 \approx 0.5.$$

Perpendicular to the grain we may consider y_0 elastic when $y_0 = 0.035$ cm. Thus: $(\alpha_{R1})_{\perp} \approx 1$.

We will use the suffix r to denote the recovery displacement. Thus $y_r = 0$ when, at the time $t' = 0$, the load has just been removed. The relative recovery creep is:

$$y_r/y_{r0} - 1 = (\alpha/\alpha_1)(y/y_0 - 1) \quad (39)$$

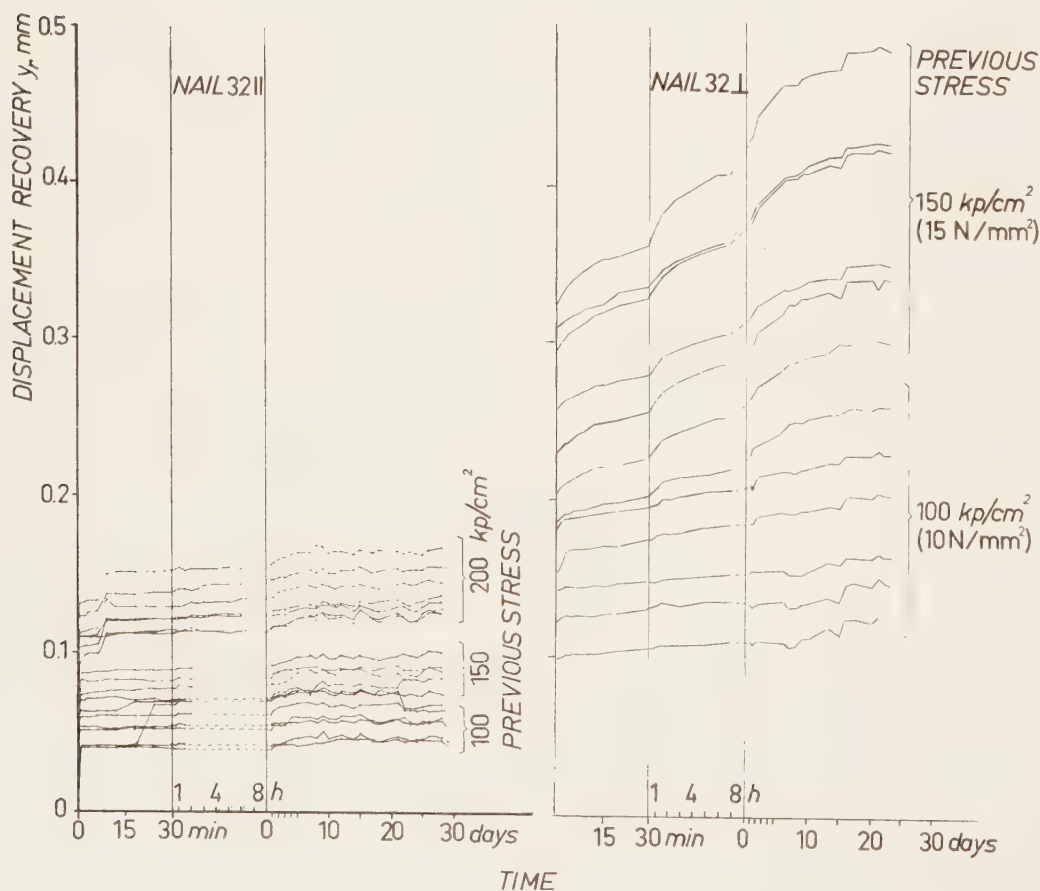


FIG. 39. Recovery in joints No. 32 nails. Cf. fig. 34.

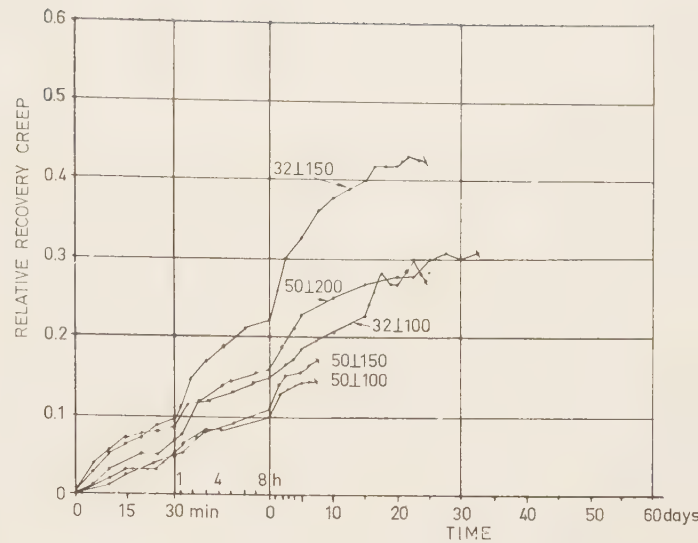
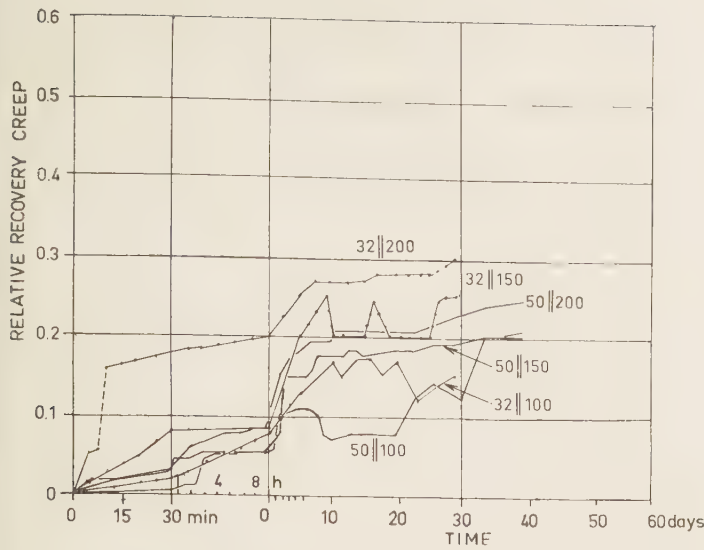


FIG. 40. Relative recovery creep after the load was removed for the first time. Each curve shows the mean value of recovery in one chain of joints calculated on the curves in fig. 33 and similar curves for joints with No. 50 nails. The irregularity of the recovery parallel to the grain (to the left) is due to the fact that the recovery approaches the error in measurement. Thus, a relative creep of 0.1 corresponds to an absolute creep of no more than 0.003 to 0.01 mm.

If $\alpha = \alpha_1$, the curves of relative creep at recovery will be identical with the curves of relative creep after loading. In reality the curves do not coincide and we may assume that the value of α is dependent of the value of i .

Comparing the creep curves in fig. 35 with the recovery curves in fig. 40 we find that the value of (α/α_1) is generally below 0.4 at loading parallel to the grain and, consequently $\alpha_{||} < 0.2$. This means that the recovery creep is so small that it can, in practice, often be neglected (cf. fig. 39). For the direction perpendicular to the grain we have above adopted the value $\alpha_1 \approx 1$. The α -value is then directly obtained as the ratio between the relative creep at recovery (fig. 40) and the relative creep at loading (fig. 35). Table 8 shows that, although the value of $\alpha_{\perp}$ is fairly constant in the day scale it logically approaches the value $\alpha = \alpha_1 \approx 1$ in the minute-scale.

TABLE 8. The recovery coefficient α for joints loaded perpendicular to the grain.

Joint	Time after loading and removing of load		
	30 min	7 days	30 days
32 $\perp$ 100	1.00	0.42	0.45
32 $\perp$ 150	0.50	0.27	0.27
50 $\perp$ 100	0.62	0.35	—
50 $\perp$ 150	0.65	0.33	—
50 $\perp$ 200	0.68	0.36	0.33

Intermittent Loading

As has already been stated the instantaneous displacement of the nails when loaded intermittently, with a period of $2T = 14$ days, approached a constant value after a few periods. As fig. 41 shows, the creep, too, approaches a value that does not seem to change much after the second period.

Also in this case, the characteristic feature is the difference in creep between the first and the following periods of loading. The classic Kelvin elements do not contribute much to this difference, since the ratio

$$(y_{3T} - y_{2T})/y_T = 1 + \exp(-2T/\tau) - \exp(-T/\tau) \quad (40)$$

never falls below the value 0.75. At high and low frequencies $T/\tau \rightarrow 0$ and $T/\tau \rightarrow \infty$ (flow) respectively, and the ratio equals 1.

In the corresponding creep elements in which the recovery is prevented, the ratio is $\exp(-T/\tau)$. Thus, with a low α -value, and provided the retardation τ_i is not long compared with the interval T , the effect demonstrated in fig. 41 is obtained.

When similar intermittent loading was applied to joints which had previously been subjected to at least one long-time creep test, the difference in creep at first loading and the following loading periods naturally disappeared.

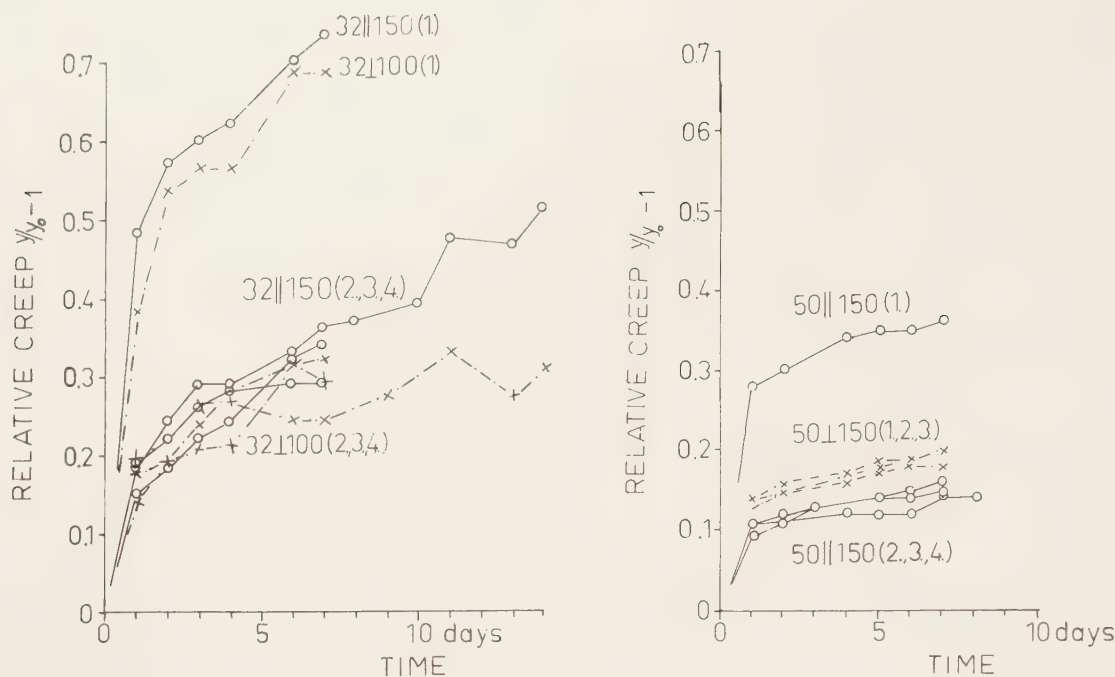


FIG. 41. Relative creep at intermittent loading, to the left in joints with nails No. 32, to the right in joints with nails No. 50. The difference in creep during the first loading interval (1) and the following intervals (2, 3, 4) is spectacular, except for joints previously exposed to long-term loading (broken curves to the right). The length of recovery intervals was seven days.

Rigid Nails under Long-Duration Load

Four of the series of nailed joints intermittently loaded as described above were finally loaded for two years. The recorded creep is given in fig. 42. The curves show the mean values obtained from the six joints in each series, i. e. the mean displacement of six loaded nails. They may be examined with reference to eq. (38).

The time function is assumed according to that of a Kelvin body:

$$\Psi_i = 1 - \exp(-t/\tau_i) \quad (41)$$

The stress function Φ_i is here for the sake of simplicity assumed to be independent of the stress (q) as well as of the time (t and i). Thus, the relative displacement, y/y_0 , will be independent of q and of the instantaneous displacement y_0 . The reference displacement defined by (35)

$$y_R = y_0 \left(1 + \sum_2^n a_i \Psi_i \right) \quad (35 a)$$

and the accompanying a_i values may then be introduced in fig. 42 and compared with the experimental curves.

The joints, however, had a loading history. They had previously been loaded to the stresses indicated during three months and were then unloaded for about one month. In our simplified reference model, we assumed the value $\alpha_1 \approx 0.5$ at loading parallel to the grain, i. e. half of the instantaneous displacement is irrecoverable. The corresponding reference curve (RD) is reproduced in fig. 42. It differs from a reference curve based on $y_0 = y_0'$ (where y_0' is the displacement at $t' = 0$ according to the figure) only on the day scale, since the irrecoverable part of the reference model participates with components with a long retardation time only, which degenerate into flow. The total flow in the model corresponds to the flow after loading for the first time to $y_0 = 2y_0'$. The figure indicates that the creep observed when 5 mm nails were loaded parallel to the grain may well be described by the reference curve thus constructed.

The reference curve given together with the curves obtained at loading perpendicular to the grain is based on the assumption that $\alpha_1 = 1$. This is correct in so far as the value of y_0' actually was close to y_0 , i. e. the instantaneous displacement was reproduced at second loading. On the other hand, the creep was partly irrecoverable, which contributes

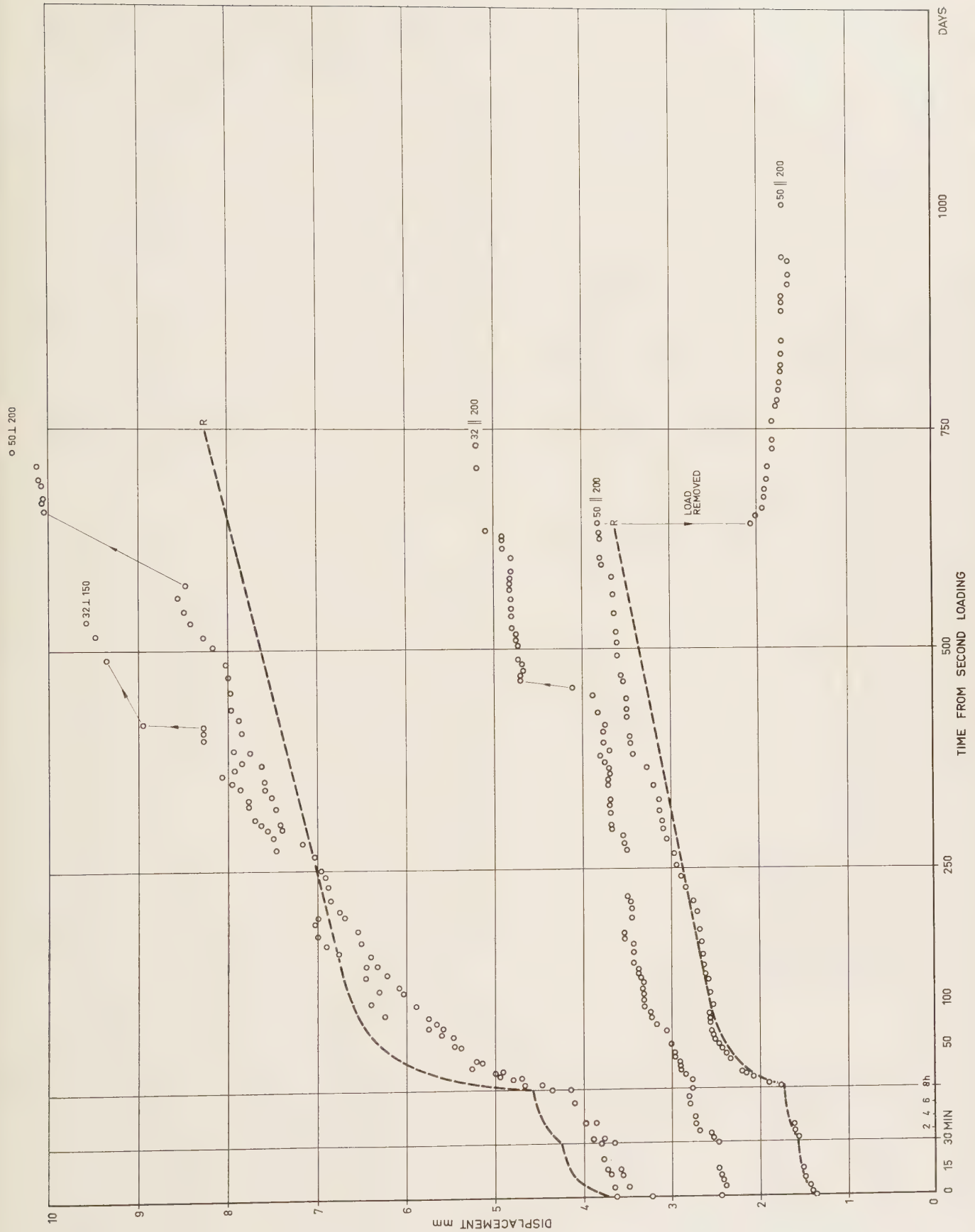


FIG. 42. Long-term displacement creep (no bending of the nails). The joints had previously been loaded to the same stresses during three months and were then unloaded during about one month. The reference curves (R) are calculated according to (35) with due regard to the loading history of the joints.

to a more slowly retarding creep than the reference creep. The results, moreover, indicate that the long-term creep is greater than shown by the reference curve. If the relative creep is assumed to be proportional to γ_0 , cf. (33), we might—since the reference curve is based on $\gamma_{0R} = 0.17$ mm—expect a flow about twice as large as in the reference model (dot-and-dash line in fig. 42). This coincides reasonably well with the long-term creep in the 50 ± 200 joints.

Application of the Results

A full treatment of how the results can be applied to nailed joints of different design and exposed to different conditions requires a separate paper. Here only a few points will be mentioned in addition to what is said in a paper by the author already re-

ferred to (Norén, 1962) and in the following sections.

The results are applicable to joints in which the curvature of the nail is small in comparison with the lateral displacement. However, tests of joints described in the third section (p. 51) indicate that the results can be used as an approximation, even if the elastic deformation of the nails is large. Still, the results should not be directly applied when plastic deformations appear in the nail.

As regards factors such as the shape of the cross section and the diameter of the nail as well as the density and moisture content of the wood, it is reasonable to assume that *the relative creep* is much less influenced by them than is the instantaneous deformation. This means that much of the deviation due to the material and the design of the nailed joints can be studied by short-term tests.

SLIP IN NAILED JOINTS UNDER SHORT-TERM AND LONG-TERM LOADING

Introduction

In the preceding sections the basic case, evenly distributed stress in the timber along the nail, corresponding to parallel displacement of the nail without bending or torsion, was considered. The general case, though, is a joint in which the ratio of the timber thickness to the nail diameter is comparatively high, and in which the compressive stress in the timber will consequently vary along the nail. In the following an account is given of tests made by the author, which supplement those previously described and facilitate the comparison with inter alia results obtained by Brock, 1965.

Instantaneous Slip at Loading

The deformations of nails and bolts in timber and the corresponding slip in the joints have been calculated by several authors, either by means of the elasticity theory or by empirical formulas (see Norén, 1962 and literature cited there). However, whichever method is used, the displacement of the nail at the juncture in a timber joint can be generally expressed by

$$y_0 = \psi \frac{4 u S}{K} \quad (42)$$

(The index 0 indicates displacement at the juncture and y_0 may here include creep).

In (42) S denotes the force per nail and member and K a displacement modulus. The coefficient of elasticity u is defined by $u = \sqrt[4]{\frac{K}{4EI}}$ where EI is the bending resistance of the nail. The factor ψ , in the following referred to as "specific displacement", has different values on either side of the juncture, ψ_1 and ψ_2 respectively. The slip between the members of the joint is

$$g = (\psi_1 + \psi_2) \frac{4 u S}{K} \quad (43)$$

When the elasticity theory is applied, $y_0 = y_{x=0} = D$ is a constant in the equation for a beam supported on an elastic foundation:

$$y = A \sinh ux \sin ux + B \sinh ux \cos ux + C \cosh ux \sin ux + D \cosh ux \cos ux \quad (44)$$

As value of K a constant value is introduced, at very low q values $K = K_H$, at somewhat higher q values $K = K_{sec} = q/y$, as is shown in the following (table 9). The specific slip in the joint, $\psi_1 + \psi_2$, calculated according to the elasticity theory, is in figs. 43 and 44 given as a function of the "slenderness ratio" um for three types of single-shear and double-shear joints. At high slenderness ratios ($um > 6$) the specific slip approaches unit. At low slenderness ratios it asymptotically approaches a hyperbola:

$$\psi_1 + \psi_2 = \frac{r_1}{um} + \frac{r_2}{um} = \frac{r}{um} \quad (45)$$

In this case (43) may be written

$$g = \frac{4r_1 S}{mK} + \frac{4r_2 S}{mK} = \frac{S}{l_1 K} + \frac{S}{l_2 K} = \frac{S}{l_e K} \quad (46)$$

The "effective length" $l_e = m/4r$, introduced by Johansen, 1941, is here called the equivalent length of the nail. If a rigid nail of the length l_e is loaded laterally by S , this will according to the definition of K result in a displacement of the nail equivalent to the slip in the joint. For a single-shear joint in which the thickness of both members is $l = m/2$ (case C, fig. 43), $r_1 = r_2 = 2$ and the equivalent

TABLE 9. Load P and instantaneous slip g in joints according to figs. 46 a and 47 a.
(Mean values from four joints.)

$P = 2 S$, kp	200	300	400
$g = 2 y_0$, cm	0.05	0.12	0.275
σ , kp/cm ²	145	260	400
$K_{sec} = \sigma d / y_0$, kp/cm ²	2900	2170	1450
$K^1 = 2 u P / g$, kp/cm ²	5000	2600	1300

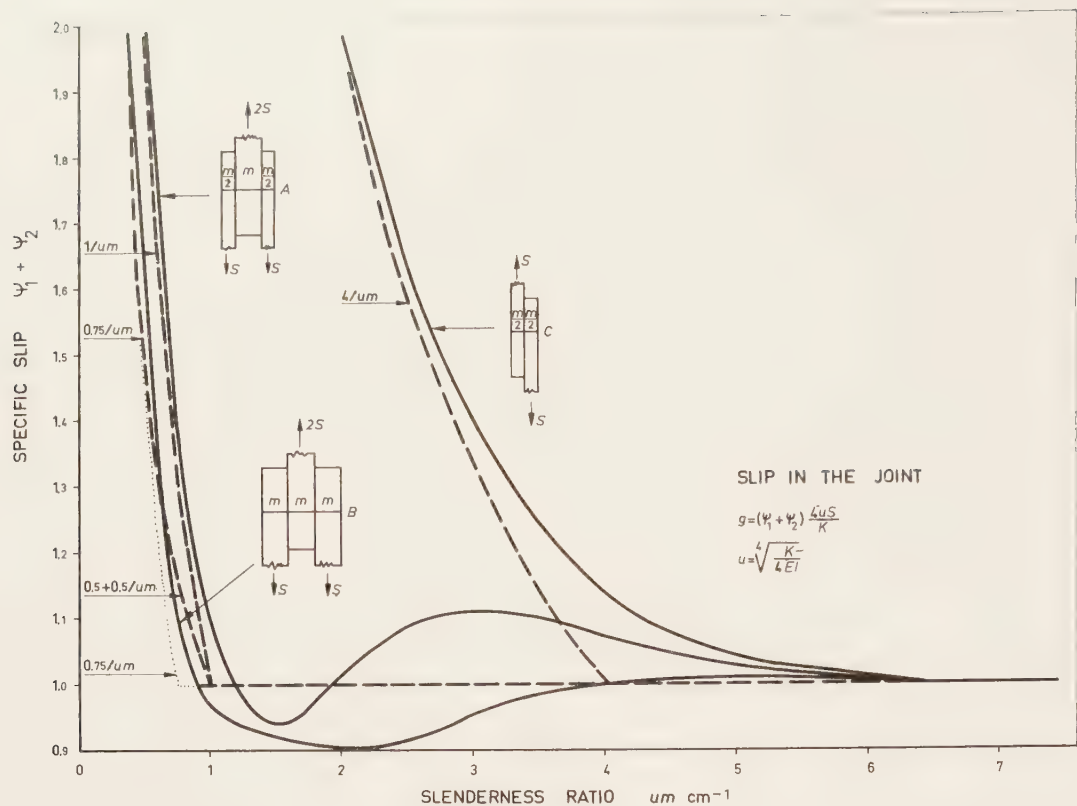


FIG. 43. Slip in nailed joints according to the elasticity theory (continuous curves) and according to an approximation (broken curves).

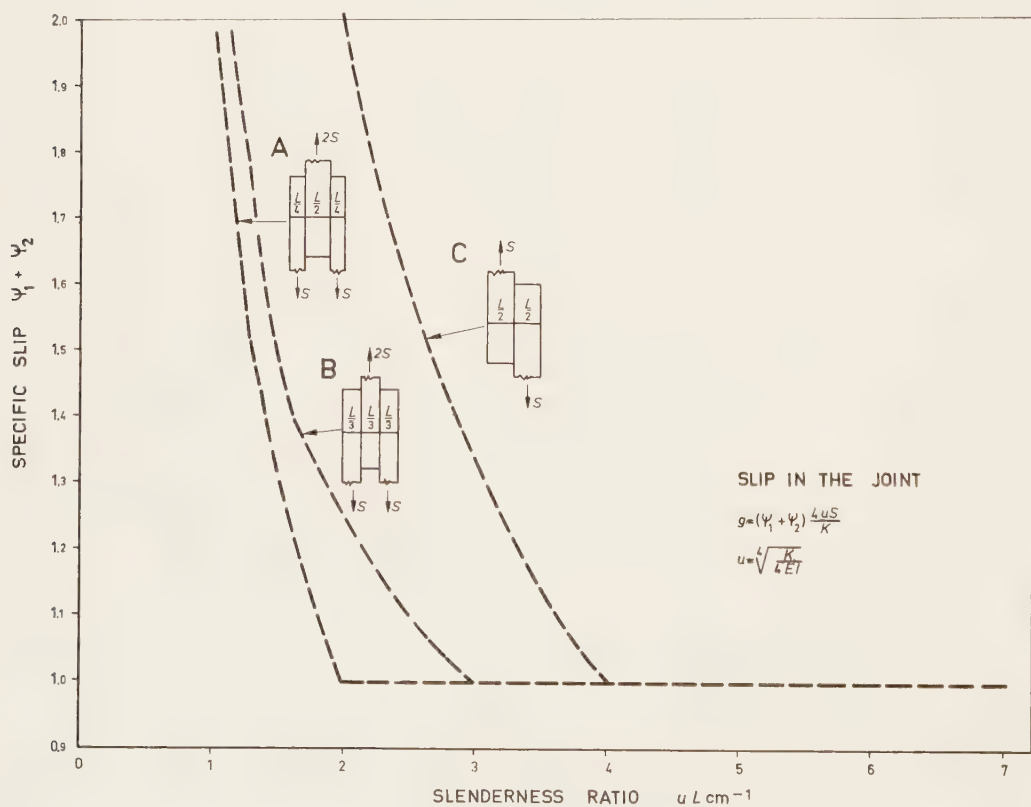


FIG. 44. Approximate slip in joints as a function of total timber thickness (L) = length of nail, cf. fig. 43.

length $l_1 = l_2 = m/4r = m/8 = l/4$. This means that the nail, without significant bending, turns round a centre situated at a distance from the juncture equal to $2/3$ of the timber thickness, fig. 45.

The method to determine the slip in nailed joints on the basis of one of the marginal cases, very low or very high ratio of the timber thickness to the nail diameter (broken curves in fig. 43), gives quite satisfactory accuracy in view of the uncertainty of the value of K .

As a complement to the determination of creep in timber under nails at evenly distributed load, the author performed creep tests on double-shear joints with a comparatively high ratio of timber thickness to nail diameter, fig. 46. In the first series, every joint contained one and in the second series four nails. In both series the thickness of the central member was 60 mm and of the side members 40 mm. The nail diameter was 5 mm (grooved wire nails No. 50). The timber used was pine—dry density $\rho_{0u} = 0.45$ g/cm³, moisture content approximately 12 % (constant relative humidity 65 % at 20°C).

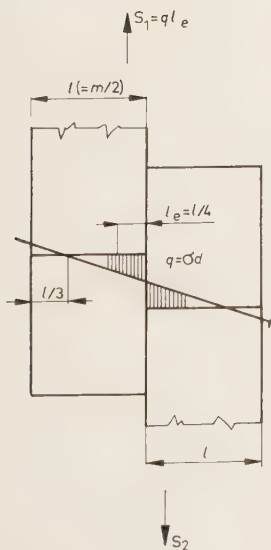


FIG. 45. Turning of the nail in a single-shear joint (case C, fig. 43).

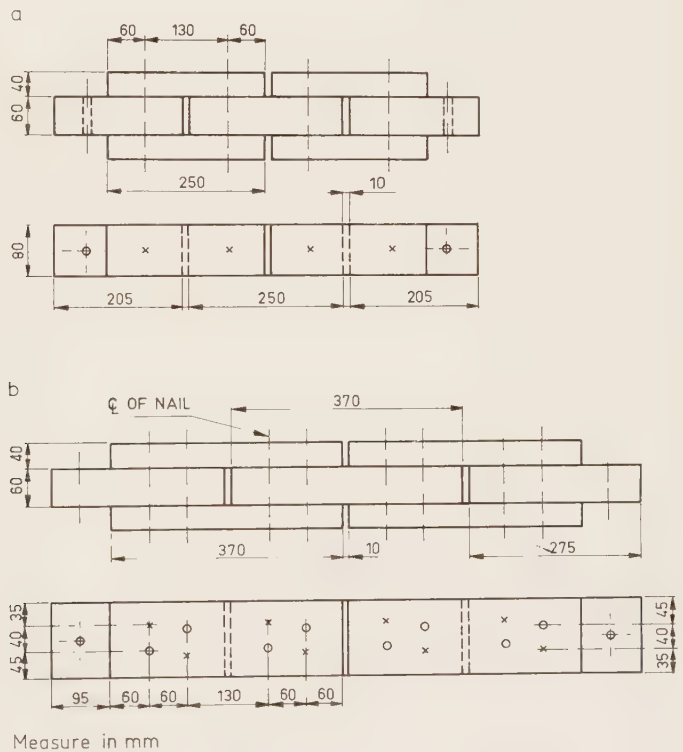
The bending resistance of the nails was $EI = 2 \times 10^6 \times 0.065 d^4 = 8100$ kp/cm² (810 N/mm²), which with $K = 5000$ kp/cm² (500 N/mm²) (cf. table 9) gives $u = 0.62$ cm⁻¹ and $um = 0.62 \times 6.0 = 3.7$. At this degree of slenderness the specific slip in the joints (an intermediate case between A and B in fig. 43) is $\psi_1 + \psi_2 = 1$, i.e. the instantaneous slip at low loads, according to (43), is

$$g = 4 u S/K = 2 u P/K \tag{47}$$

The mean value of the instantaneous slip in the joints, is shown in table 9. The displacement of the nail in the timber at the juncture may in this case be assumed to be the same in the side members as in the central member: $y_0 = g/2$. The corresponding compressive stress σ kp/cm² is calculated from (34), which for the nail and timber in question may be written:

$$y_0 = \sigma d \left(\frac{1}{6000} + \frac{\sigma}{80 \times 10^4} \right) \tag{48}$$

The values of σ and of the secant modulus $K_{sec} = \sigma d/y_0$ thus obtained are given in table 9, in



Measure in mm

FIG. 46. Long-term loaded nailed joints (double shear) with a high slenderness ratio: a) 1 nail per joint, b) 4 nails per joint.

which is also found the value (K^1) of the displacement modulus, which introduced in (47) gives the recorded slip.

For the joints under the higher loads at least, the values of K_{sec} and K^1 are of the same order. This means that it is possible, on the results of the basic investigations of the relation between the compressive stress under the nails and the lateral displacement of the nail, here represented by (48), to calculate a value K_{sec} , expressed in γ_0 , which, introduced in the expression of the elasticity theory for the relation between P and γ_0 , can be used with reasonable accuracy to determine the relation between the load and the slip in the joints tested here.

Table 10 gives a few other data on short-term deformation in nailed joints approximately coinciding with the data and conditions as above. The load, which is expressed in kp per nail and member, is in accordance with BABS, 1967.

To judge by the values in the table, a slip of 0.3 to 0.5 mm is to be expected after a short time under normal working load. A condition is that the initial friction between the members is eliminated. When nailed joints are tested in the laboratory, this is generally achieved by inserting thin washers in the joints before nailing, which are then removed before the joints are tested. This leaves a space of a few tenths of a mm between the members, corresponding to the space normally obtained in practice when the timber dries. The friction is of decisive importance for the slip under moderate loads. This is shown by the values in the last column of table 10. Granholm, 1949, obtained an instantaneous slip (load below working load) in tightly nailed joints amounting to no more than 1/5 of the slip when the friction had been eliminated as above.

The importance of the initial friction diminishes with increasing load. Mack, 1960, obtained an instantaneous displacement of 0.15 to 0.9 mm in

TABLE 10. Short-term slip in nailed joints under normal load. Air-seasoned timber. Force parallel to the grain. (Mean values from at least four joints.)

Type of nail	d mm	T	r g/cm ³	n	s mm	m mm	s mm	S_{BABS} kp	S kp	$2\gamma_0$ mm	$d/2\gamma_0$	References
Ø	4.2	Spruce	0.48	2	40	40	40	48	65	0.8		Meyer, 1957
Ø	4.1	Pine	0.445	2	30	30	30	46	34 83	0.15 0.75		Brock, 1957
∅	4.2	Pine	0.45	1	35	35	—	70	70	0.4	10.5	Niskanen, 1959
∅	5.5	Pine	0.45	1	55	55	—	105	110	0.5	11	Niskanen, 1959
∅ R	3.7	(Pine) ^a		1	22	100	22	50	40	0.32	11.6	Granholm, 1949
∅ R	4.3	Spruce		1	38	175	38	70	70	0.54	8	Granholm, 1949
∅ R	4.3	Spruce	0.36	1	50	50	50	70	65	0.35 ^b	12.3	Norén, 1959
∅ R	5.0	Pine	0.42	2	60	60	60	90	80	0.38 (0.13) ^b	13	Norén ^c
Ø	3.1	Spruce		2	22	22	22	27	37.5	0.46 ^b	6.7	Möhler, 1961
Ø	4.2	Spruce		2	35	35	35	48	84	0.63	6.7	Möhler, 1961
Ø	6.0	Spruce		2	60	60	60	100	165	0.53	11.3	Möhler, 1961

^a Timber species not mentioned. Compressive strength 375 kp/cm² (37,5 N/mm²).
^b Joints with friction between the members after ordinary nailing.
^c Non-published data.

The following symbols are used:

d	diameter of nail	s, m	thickness of members
Ø ∅	section of nail (R = grooved)	S	shear force per nail and joint at testing
T	timber species	S_{BABS}	shear force allowed according to the Swedish building code, BABS, 1967
r	dry density	$2\gamma_0 \approx g_0$	short-term slip in the joint
n	number of joints between members		

tightly nailed joints of dry, light pine (Radiata pine, moisture content 11 %) at a load 1.5 to 2.5 times the design load, and in corresponding tests without friction a displacement of 0.25 to 0.75 mm. Thus, the influence of the friction does not seem to have been very great in the tight-nailed joints. These initial displacements are very low (cf. table 10), which can only in part be explained by the fact that the nail diameter (3.6 mm) was comparatively small.

Brock, 1957, and later Mack, 1960, have derived empirical expressions for the short-duration load that will cause a certain displacement and failure in joints:

$$S = C d^p \quad (49)$$

For the failure load Brock, 1957, gives $p = 2$ and nearly the same C values for single-shear and double-shear joints, i. e. quite in accordance with current theories (Johansen, 1941). For the load corresponding to a certain slip, however, different values are given for as well p as C in single-shear and double-shear joints, e.g. a force per joint and nail which in pine (dry density 0.4 g/cm³, moisture content 18 %) gives $g = 0.015$ cm:

$S_1 = 150 d^{1.4}$ and $S_2 = 140 d^{1.25}$ kp (d in cm) in single-shear and double-shear joints, respectively. Accordingly, for thick nails, $d = 0.64$ cm, $S_1 = S_2$. For thin nails S_2 will be slightly greater than S_1 , e.g. at $d = 0.254$ cm, $S_1 = 21.6$ kp and $S_2 = 25$ kp. Still, the difference is not large enough to motivate different expressions for single-shear and double-shear joints.

It should be observed that the tests referred to above were performed on joints as in case B and C in fig. 44, thus on joints in which all members are equally thick and the total timber thickness equal to the length of the nail. The specific slip can then be expressed by the slenderness ratio uL where L is the length of the nail. For round nails

$$uL = \frac{L}{d} 0.04 \sqrt[3]{K} \quad (50)$$

With the lowest ratio $L/d = 19.2$ (2 in. S.W.G. gauge 12) and $K = 0.75 \times 5000$ kp/cm² (500 N/mm²) we obtain $uL = 6$. According to fig. 44, this is the marginal value above which the slip, calculated according to the elasticity theory, should be the same in single-shear and double-shear joints at equal load per joint.

Creep in Joints with Slender Nails

When creep is discussed, the concept's *relative load*, the ratio of the absolute load to the short-duration breaking load defined in a certain way, and *relative deformation*, the ratio of the total deformation after a specified loading period and the deformation after loading during a given shorter period, often defined as "instantaneous deformation", are generally introduced. Timber as structural material is in practice regarded as a, in the first approximation, solid material without time-dependent deformations. The deformation as a function of the short-duration load serves as a basis for the dimensioning calculations and has been fairly well determined. Consequently, the short-term deformation is in itself a measure of the load and the above defined relative load may be considered of a secondary importance.

Also, in dealing with timber joints, it is suitable to use the relative measure of the total deformation or creep based on the short-term deformation. A condition is, of course, that the short-term deformation is actually representative of the load and not influenced by an "initial" deformation, which may be positive, i. e. occur in the form of an "adjustment deformation" (permanent set), or negative as in the case of initial friction. Particularly in timber joints with mechanical fasteners, there is a risk that such irrelevant deformations get included. In nailed joints, the friction between the members can, as has been mentioned, give a considerably reduced initial deformation. This will exaggerate the relative value of the creep, provided that the friction diminishes as the load is increased or in the course of time.

Examples of very small initial deformations and correspondingly high values of relative creep are given by Brock, 1965. In double-shear joints an instantaneous slip of about 0.003 in. (0.076 mm) was obtained at a load of 150 kp (1500 N). This is a remarkably low value compared with the values found by the author, 0.2–0.3 mm according to figs. 46a,b. Pine with approximately the same density and moisture content was used in both cases. The bending resistance of Brock's round 5.4 mm nails was practically the same as that of the 5 mm grooved wire nails used by the author. If we assume that also the value of K was the same in both cases, the slenderness coefficient will be $um > 1$ and the displacement might also be expected to be the same in both cases.

Fig. 47 shows the creep in the joints according to fig. 46 a, i.e. with one 5 mm nail in each joint. The nailed joints were placed in a recycle system of air passing over a saturated water solution of NaNO_2 . Above all owing to variations in the difference between the temperature of the salt bath and the temperature of the air around the joints, small variations in the relative humidity of the air (φ) were obtained. During a "normal 24-hour period"

$\varphi = (66 \pm 1) \%$ but over the whole period the variations were slightly greater: $\varphi = (66 \pm 3) \%$. The creep on the whole agrees with the reference creep (R) obtained when the stress was evenly distributed along the nail (35). It should be observed, however, that the R function in the latter case only represented the creep when the instantaneous deformation was $y_0 = 0.17 \text{ mm}$, since the relative creep increased with the value of y_0 . It was even stated

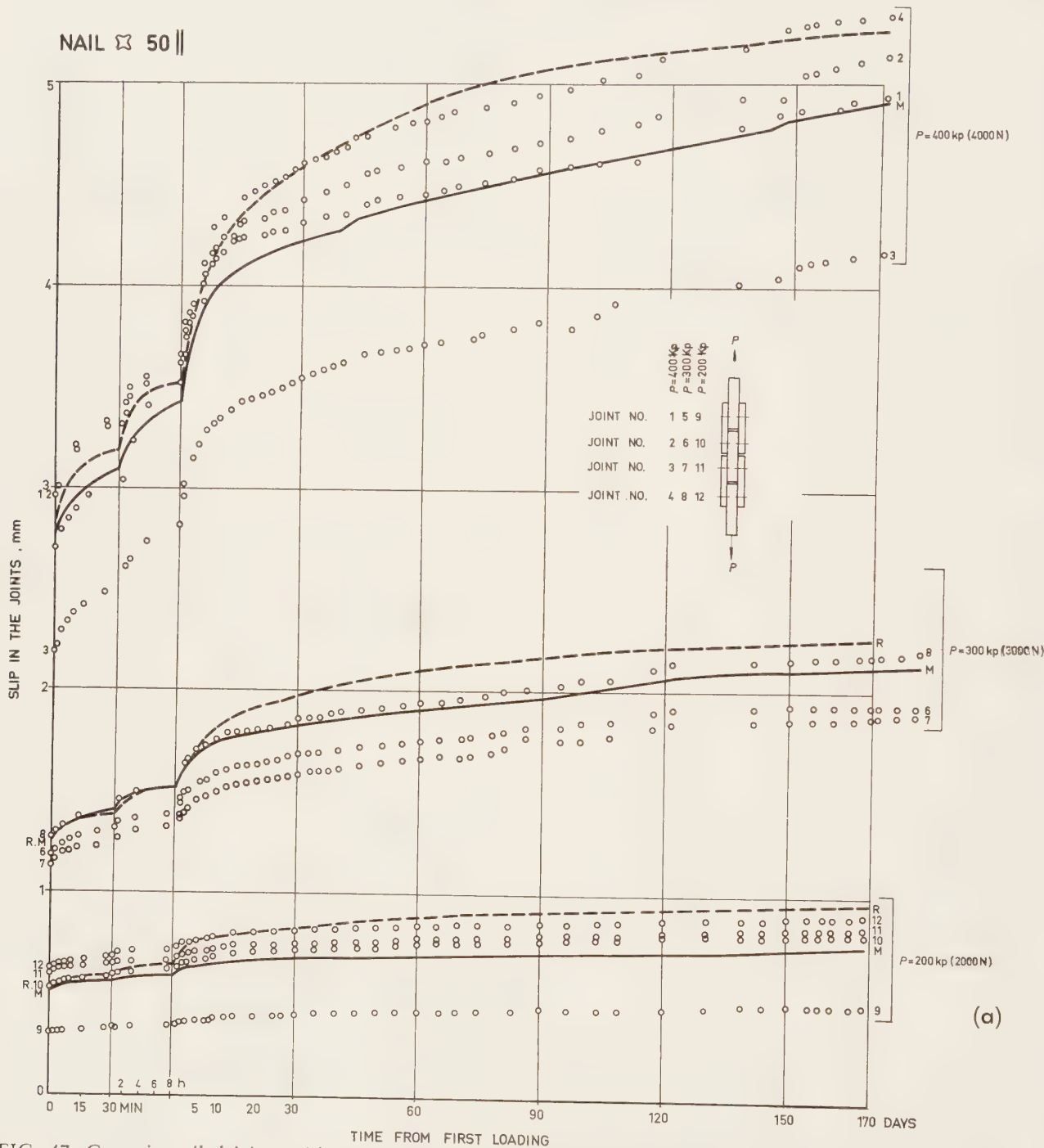
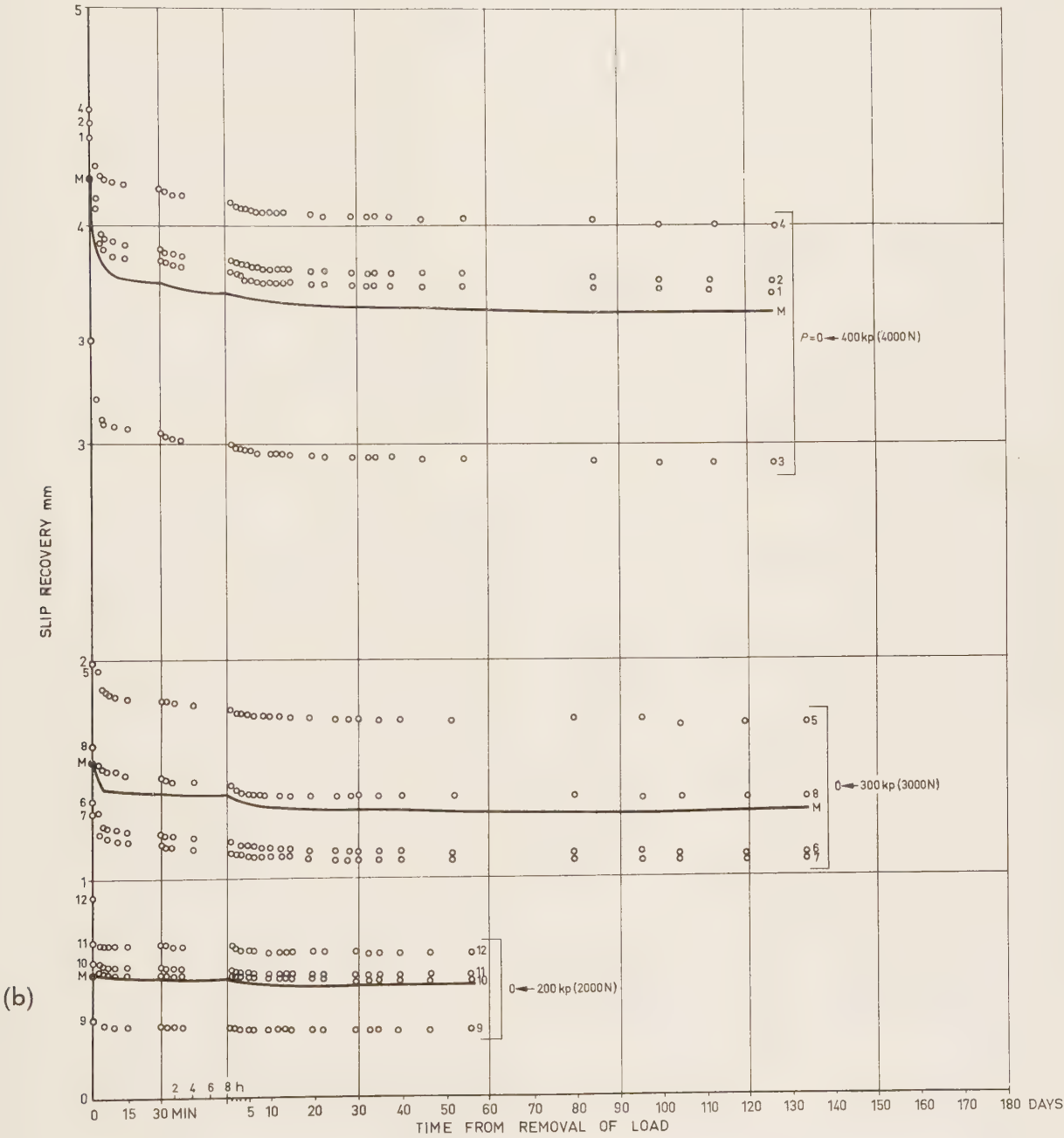


FIG. 47. Creep in nailed joints with one nail No. 50 per joint, fig. 46, a. Above (a): Slip at first loading. To the right (b): Recovery slip at first unloading. Continuous curves (M) = Mean value of creep. Broken curves (R) = Creep of reference body.

(second section, p. 34) that it was possible, as a first approximation, to assume proportionality between the relative creep and γ_0 . This does obviously not apply when the nail bends under the load. Otherwise, the creep would have been 50 % greater than indicated by the R curve even in the series of joints to which the lowest load was applied, where the instantaneous displacement at the juncture was $\gamma_0 = 0.5 g_0 = 0.255 \text{ mm}$, and in the series under

greater load, the creep would have been several times greater than the measured creep. The bending of the nail has—as has earlier been shown theoretically (Norén, 1962)—retarded the creep. The agreement with the R curves, based on $\gamma_0 = 0.17 \text{ mm}$ at all the time scales to 6 months, must be regarded as a coincidence in so far as the results cannot be applicable to all slenderness ratios. (Possibly, the limits indicated by the intersection point



of the broken curves in fig. 43 could be used pending a more developed theory. This means that the creep would be regarded as proportional to γ_0 , i.e. the relative creep independent of γ_0 , when the specific slip in the joint is $\psi_1 + \psi_2 = 1$, and would be assumed to grow more than proportionally to γ_0 , according to the displacement tests, when the specific slip is proportional to $1/um$.)

Table 11 shows the creep obtained by Brock in joints consisting of three 43 mm thick pine members (*Pinus Silvestris*) with one round nail, 5 S.W.G. ($d = 5.4$ mm), in double-shear. The values in the table converted to mm are averages for four joints loaded to about 150 kp (1500 N). Although the relative slip is very great, the creep in mm is moderate.

After one year the creep is 0.235 mm and after six years 0.565 mm. The mean curve in figs. 47 a,b for a load of 200 kp (2000 N), (10% above the maximum allowable long-term load according to BABS 1967) gives, if extrapolated to the slope of the R curve, a creep of 33 mm after one year and 0.91 mm after six years, i. e. a creep exceeding the creep in Brock's joints by 40 and 60 %, respectively. Since the load was 30 % higher, the agreement must be regarded as good.

The difficulty of foreseeing as well the short-term deformation as the creep, in the presence of friction and variations in the moisture content, is exemplified by Möhler's values, table 12 (Möhler, 1961). The short-term deformation in the two joints, series 1, is for unknown reasons exceptionally great, 20—30 times greater than the values in table 11. Also the creep is great in series 1 and 2, but in series 4 the values are initially smaller than according to the reference function (R). The relative air humidity varied considerably during the loading period and the timber was exposed to drying. Shrinkage alters the embedding conditions, the friction and the geometry of the joints (space between the members). To this should be added that the combined effects of the rheological properties of the timber and sorption have, in all probability, increased the slip.

The creep test with one-nail joints (fig. 46 a) was followed by a similar test with four-nail joints (fig. 46 b). Nails and timber were closely matched in both

TABLE 11. Creep in double-shear joints 5 S.W.G. nails, according to Brock, 1965.

Time, years ...	0	1	2	4	6
Instantaneous slip, mm	0.076	—	—	—	—
Total slip, mm	0.076	0.31	0.39	0.58	0.64
Relative slip	1.0	4.08	5.17	7.67	8.42

TABLE 12. Creep in nailed joints—pine, double shear—with round wire nails, $d = 4.2$ mm, according to Möhler, 1961. Timber thickness 35 mm. The allowable working shear load on the nail is 63 kp (630 N) according to DIN 1052, i.e. as in series 4, and 48 kp (480 N) according to BABS 1967.

Series	S kp	g_0^a mm	g_{15} mm	$g/g_0 - 1$		
				15	60	180 days
1	84	2.38 1.76	4.05 3.0	0.7 0.7	1.65 1.1	2.6 2.0
2	86	0.52 0.45	1.7 1.6	2.3 2.5	3.2 3.6	3.9 4.2
4	63	0.52	0.61	0.2	0.45	1.4
Reference funktion (R)				0.55	0.75	0.87

^a $g_0 \approx 2\gamma_0$, cf. table 10.

series and the same load per nail was applied. However, during the testing of the second series of joints, the relative air humidity occasionally varied.

The displacement at the application of the load and the creep are shown in fig. 48. The instantaneous displacement is somewhat smaller than in the series with one nail per joint. The relative creep, on the other hand, is slightly larger but follows the R function satisfactorily during the first month. The short-term creep in the series under the highest load is comparatively great, but this may to some extent be due to the variations in the relative humidity. The increase in the creep after loading during one month can presumably be ascribed to the fact that the air humidity increased during about one week. As other experiments have shown (Eriksson & Norén, 1965), the first absorption to a certain moisture content level, or desorption, will increase the deformation of wood under stress. After 65 days the relative air humidity was $(66 \pm 2) \%$, at which humidity the creep curves resumed the shape corresponding to the R function.

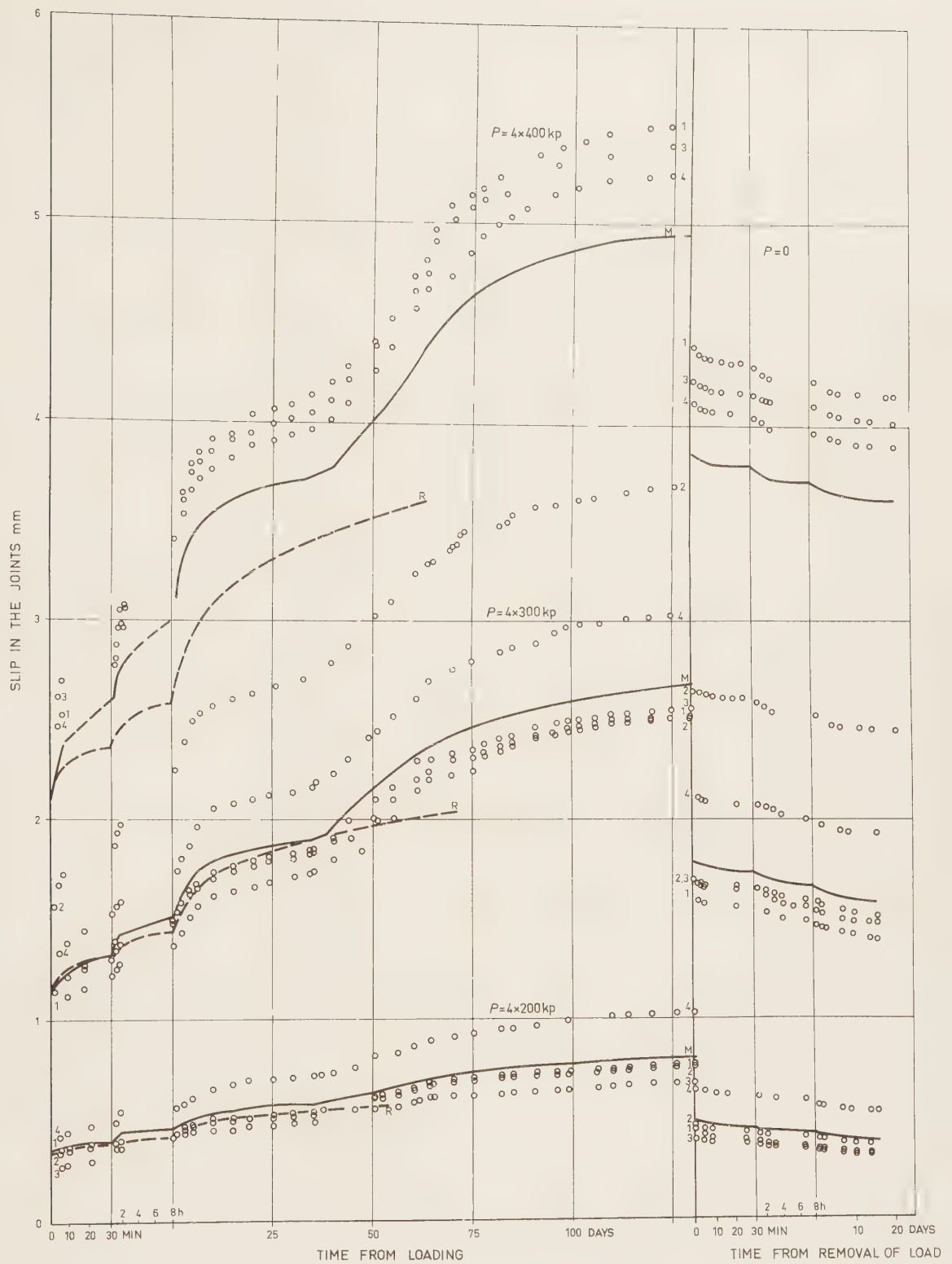


FIG. 48. Creep in nailed joints according to fig. 46, b. Four nails No. 50 per joint. (Cf. fig. 47.)

Slip Recovery

The instantaneous recovery in the joints, fig. 46 a after unloading is given in table 13.

The ratio g_{r0}/g_0 in table 13 is here identified as the coefficient of recovery α_1 in (38). With reference to the results obtained by testing joints with rigid nails (second section, p. 34), the value of this coefficient should be $\alpha_{R1} \approx 0.5$ at loading parallel to the grain. However, in a joint with elastically bending nails (short time of retardation), the value of α_1 might be expected to be higher than this reference value. Only at loads so high that the nails are plastically deformed, should the value of α_1 be lower than according to the displacement tests with straight nails. As table 13 shows, different values of α_1 were, for some unknown reason, obtained in joints with one nail and in joints with four nails (values in brackets).

The recovery creep curves are reproduced in fig. 47 b. Table 14 shows the ratio of the creep recovery to the creep 8 h and 30 days after the change of load (application of load and unloading)

alpha prime = (g_r - g_r0) / (g - g_0) (51)

alpha prime is equivalent to the mean value of alpha for a series of bodies (fig. 49) with a creep according to (38). On the basis of the values of alpha_1 in table 13 and of alpha prime in table 14 for joints with one nail, it seems reasonable to accept, for approximate calculations, a constant alpha_i = alpha = 0.40, corresponding to a permanent slip in the joint (g - g_r)/g = 0.60.

However, the deviation of the alpha_1 and alpha prime values from the 0.40 value at the load 400 kp (4000 N) per nail is rather great. This may to some extent be due to the difficulty of determining an accurate value of instantaneous slip at this high load.

The recovery coefficient 0.40 agrees approximately with the recovery in Brock's joints under long-duration load (Brock, 1965), which was 0.28—0.42 of the total deformation. Mack, 1963, in Radiata Pine joints, obtained a recovery of, on an average, 0.25 of the total deformation, a recovery of which the principal part was obtained immediately after unloading.

TABLE 13. Instantaneous recovery g_r0 at unloading of P. Joints according to fig. 46 a. (Mean values from four joints.)

P = 2 S, kp ...	200	300	400
g_r0, cm	0.020	0.059	0.069
K_r = 2 u P/g_r0, kp/cm^2	17,100	6900	8300
g_r0/g_0 (cf. table 9)	0.40 (1.0)^a	0.49 (0.75)^a	0.25 (0.54)^a

a Joints with four nails as in fig. 46 b.

TABLE 14. Recovered creep versus creep alpha prime, see (51), and non-recovered creep in the joints, figs. 46, a, b.

Time after application of load and unloading	Joints with one nail. Load per nail (2 S) kp			Joints with four nails. Load per nail (2 S) kp		
	200	300	400	200	300	400
8 h	0.45	0.68	0.82	0.55	0.35	0.16
1 month	0.40	0.45	0.41	0.50	0.34	0.16
(g - g_r)/g	0.65	0.61	0.73	0.46	0.57	0.73

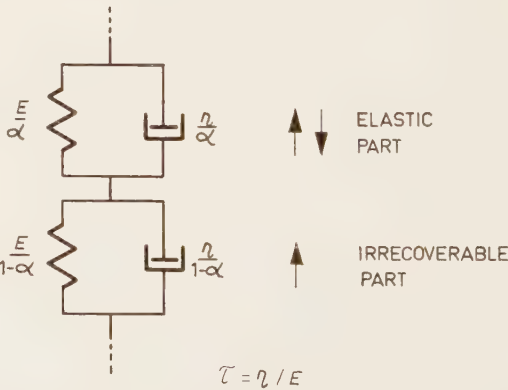


FIG. 49. Pair of unidirectional and fully elastic Kelvin bodies for the illustration of incomplete elasticity. A full elasticity (alpha = 1) the pair functions as one body with the parameters E and eta.

THE SLIP MODULUS IN NAILED JOINTS

Introduction

Even under moderate loads, the slip in joints with mechanical fasteners will often be of importance for the behaviour and strength of the jointed timber structure. This is a well-known fact (see e.g. Granholm, 1949, 1961 and Norén, 1959) to which, however, sufficient attention is not always paid in static calculations. One reason is that it is difficult to give an adequate representation of the displacements occurring in timber joints, since they depend on a number of different factors. In nailed joints alone the displacements vary with a number of factors, such as the type of nail (certain fasteners may be considered as a group of nails), the design and, above all, the properties of the timber. Much of this has been the subject of basic as well as specialized research which, owing to lack of or insufficient analysis of the importance of the experimental conditions, often has given results of limited validity.

In the first section (p. 9) the modulus K was introduced to define the ratio of the stress on a rigid nail to its displacement in the timber. The modulus K was used in the same sense as the elasticity modulus E , defined as the quotient of stress and strain below the proportional limit. Thus, K as well as E are material constants. Though the E values for wood found in the relevant literature should be slightly dependent on the speed of loading at their determination, the influence of time is small and generally neglected.

In many materials proportionality between stress and strain does not exist, or the proportional limit is very low. The elasticity modulus as defined above, representing the slope of the stress-strain curve at origin, will then give insufficient information on the deformation for most purposes. However, many authors, in dealing with materials of non-Hookean behaviour, use the elasticity modulus E (and corresponding elasticity coefficients) merely to express the ratio of stress to strain (secant modulus) or its derivative (tangent modulus),

thus getting E values which may be very much influenced by stress as well as by time. This also applies to the treatment of deformations in nailed joints, which show a spectacularly curved load-slip relation, especially when the ratio of the timber thickness to the nail diameter is high.

The Slip Modulus

The relation between the force per nail and the slip in the joint is according to (43)

$$k = S/g = \frac{K}{4u} \frac{1}{\psi_1 + \psi_2} = \frac{K}{4u\psi} \quad (52)$$

The factor k (kp/cm) is in the following generally called *slip modulus* and is related to each individual nail in the joint, i.e. S = force per nail and joint.^a

If the spacing of the nails in the direction of the force S is denoted a and the width of the surface at the juncture b , the shear stress in the joints is

$$\tau = \frac{S}{ab} = G_r \frac{g}{r} \quad (53)$$

In (53) r is the distance between the central lines of the members and $G_r = k \frac{r}{ab}$ (kp/cm²) may be defined as a shear modulus equivalent to the slip in the joints. If n_1 rows of nails can be fitted into the width b , we obtain $a = a'n_1$ where a' is the distance between the nails in each row.^b

^a S and k can be alternatively related to any arbitrarily chosen fastener or group of fasteners (group of nails). For the calculation of jointed structures, the slip modulus is sometimes generalized to apply to slip between two from each other separated timber members. In this case the slip referable to the junctures forms part of the resulting slip.

^b G_r must not be confused with the specific slip modulus $G = k/a$ (kp/cm²) used by Pleskov (according to Niskanen, 1961). The displacement modulus K introduced by Johansen, 1941, which has here been used should not be confused with Granholm's specific displacement modulus $K = k/ab$ or Niskanen's collective displacement modulus for n nails $K = nk$ (cf. footnote a). Johansen's $k = K/d$, where d is the diameter of the fastener, is of course not equal to the slip modulus k .

The slip modulus k has a great influence on the stiffness of jointed units and thus as a rule on their strength. It is in this connection important to distinguish between the true and the *apparent* value of the slip modulus. The true value may be defined as the value resulting from tests on timber joints and is equivalent to a material property. The apparent value of the slip modulus is the value that must be used when the deformation or the strength depending on the deformation of a timber structure is calculated in order to make the calculated result coincide with the observed deformation or strength. The difference between the apparent and the true slip modulus is a measure of the realism of the assumptions (theory) on which the calculation is based. One important approximation generally introduced in order to facilitate the statics of jointed timber structures is that the slip modulus is assumed to be independent of the size and the duration of the load. The result is that the value of the apparent slip modulus will differ for joints of the same type in different structures and also in the same structure at different loads.

The question of the apparent value of k is in fact analogous to the question of finding an (apparent) value of K to be used in the expression of the elasticity theory for the calculation of the actual displacement in a nailed joint. However, most of this falls beyond the scope of the present report, though the apparent slip moduli determined by different research workers are of interest.

In (52) $\psi = 1$, if the nail is sufficiently slender. In joints with one round wire nail ($E = 2 \times 10^6$ kp/cm²) the slip modulus will then be

$$k_{\phi} = 6.3 K^{3/4} d \quad (54 a)$$

(k in kp/cm, K in kp/cm², d in cm), and for one grooved wire nail

$$k_{\square} = 6.73 K^{3/4} d \quad (54 b)$$

According to fig. 43 the slenderness requirements are for double-shear joints in which the thickness of the side members is at least half the thickness m of the central member

$$m/d_{\square} \leq 1/ud_{\square}$$

and for single-shear joints with a timber thickness of at least l

$$l/d_{\square} \leq 2/ud_{\square}$$

If K is independent of d , the slip modulus k will, according to (54), be directly proportional to the diameter of the nail d , i. e. with $K = 5000$ kp/cm² (500 N/mm²) (cf. table 9)

$$k_{\phi} = 3700 d_{\phi} \text{ and } k_{\square} = 4000 d_{\square} \quad (55)$$

It has on an earlier occasion been suggested by the author that the former value be applied when the load is equal to the working load of the nail (Norén, 1955). Niskanen, 1961 and 1964 (Tränormkommiten) has also suggested a diameter-dependent slip modulus of the same order ($k = 3500 d$) for the calculation of the strength of composite columns assembled by nailing.

It might seem as if the ratio of the stress to the displacement, $K/d = \sigma/y$, and not K ought to be independent of the diameter, at least at loading parallel to the grain. Results pointing in that direction have also been obtained for heavy fasteners in prebored holes (Johansen, 1941). Some of the displacement tests, figs. 24 and 25, carried out by the author on pine with both round and grooved wire nails ($d = 0.21, 0.35$ and 0.51 cm) also gave a diameter-dependent value of K parallel to the grain, cf. (3):

$$K = C(2d + 0.3) \quad (56)$$

Furthermore, the diameter could not be proved to influence the stress that caused a displacement the non-Hookian part of which was $0.02d$. The corresponding secant modulus

$$K_{\text{sec}} = \frac{K}{1 + 0.5(2d + 0.3)} \quad (57)$$

thus seemed to be less influenced by the diameter than K . No diameter dependence of K perpendicular to the grain was proved, neither in pine nor in spruce.

Table 15 shows some values of slip in nailed joints obtained by Granholm, 1949 and 1961. The slip modulus k_s is the secant modulus from zero load to the working load specified in BABS 1967. The

TABLE 15. Slip moduli of nailed joints according to Granholm, 1949 and 1961. Secant modulus (k_s) at working load per nail (BABS, 1967).

d cm	k_s kp/cm	k_s/d kp/cm ²	Remarks
0.16	2150	13,400	
0.16	1000	6,300	Loosely nailed
0.16	1290	8,000	No friction (space between members)
0.22	5200	24,000	Tightly nailed
0.28	1640	5,900	
0.31	1920	6,200	
0.31	2250	7,300	Partly across the grain. Friction
0.31	600	1,900	Partly across the grain. No friction
0.33	1650	5,000	
0.37	2100	5,700	
0.43	1300	3,000	
0.51	7900	15,500	Friction
0.51	2100	4,100	No friction
0.51	700	1,400	Irrelevant movements cannot be excluded owing to extreme type of test specimens
0.80	650	800	

TABLE 16. Slip moduli of uniformly nailed I-beams and box girders at bending tests according to Möhler, 1953. The values refer to a load per nail equal to the working load according to DIN 1052 (round wire nails). Mean value and in brackets lowest value.

Nail diameter d cm	Number of tests	Slip modulus k kp/cm	k/d kp/cm ²
0.31	11	1570 (1130)	5000 3650
0.28	8	1660 (1075)	5700 3840

TABLE 17. Slip moduli of nailed beams at bending tests according to Granholm, 1949 and 1961. Load approximately corresponding to working load according to BABS 1967.

d cm	k kp/cm	k/d kp/cm ²	Remarks
(0.22)	(2810)	(12800)	Vierendeel girder at 50 % of working load per nail
0.31	600	1900	{ Panel beam. No friction. Force partly across the grain
0.43	2250	5200	
0.43	1300	1300	I-beam
0.37	400	1100	Laminated beam. No friction (space between layers)

TABLE 18. Time dependence of the slip modulus according to eq. (36)

Time after application of load	$k/k_{0.5}$	k/d kp/cm ²
0	1.27	3400
12 h	1	2700
7 days	0.915	2500
1 month	0.78	2100
1 year	0.63	1700
10 years	0.295	800

values refer to grooved wire nails and the moduli are “true” moduli based on tests on joints (cf. table 9). The conditions, however, varied from case to case, particularly with respect to the friction between the members. The values of k_s and k_s/d thus vary considerably. The coefficient of variation is practically the same for both factors and no conclusions as to the influence of the diameter are possible.

Some slip moduli for nailed joints calculated by Möhler and Granholm on the basis of deformations in nailed beams (apparent moduli) are given in tables 16 and 17. These values also vary considerably. Summing up the available test results, it does not seem unreasonable to base the slip modulus for joints with grooved wire nails under short-duration load, not exceeding the working load according to BABS 1967, on $K = 3000 \text{ kp/cm}^2$ (300 N/mm^2), applicable to air-seasoned pine and spruce irrespective of the direction of the load. Calculated according to (54b), which presupposes a slenderness ratio of at least $m/d = 3.65$ in double-shear joints and of $(m/2)/d = 7.3$ in single-shear joints (timber thickness according to fig. 42), this corresponds to the slip modulus

$$k = 2700 d \tag{58}$$

The value is given primarily as “true” value but may also be used as basic value for deformations in bending in built-up nailed structures.

In order to estimate the value of k at long-duration loading, we will use the secant modulus of the earlier suggested simplified reference curve for creep (36)

$$\gamma/\gamma_{0.5} = k_{0.5}/k = 1 + 0.385 [1 - \exp(-t/24)] + 0.055 \times 10^{-2} t \tag{59}$$

Here t is the time in days beginning 12 h after the application of the load, i. e. the time when value of the secant modulus was $k_{0.5}$. We will here assume that the value chosen as above, $2700 d$, corresponds to the secant modulus after 12 h of dead load. Table 18 shows how the secant modulus decreases under prolonged load.

This leads us to the question of which t values are to be regarded as representative of “short-duration” and “long-duration” load, respectively. It is difficult to find a general answer. For ordinary building

structures 12 h is an extremely short period, since it is necessary to consider the accumulated effect of repeated loading. However, wind load of high intensity might be one example. Other live loads classed as short-duration loads in the building code may motivate lower k values. Loads of longer duration than 10 years are not included in the table. Only when the dead load (constant load) predominates very much, which is rare in timber structures, will the total duration of loading during the life of the structure be 10 years or more.

It is of course possible in static calculations to introduce a slip modulus depending on the force. Cederwall, 1961, has presented a fairly simple method of designing nailed joints loaded by moment to be used when the slip modulus is decreasing with increasing load. He also applies differential calculations to composite columns in which the joints have a non-linear load-slip curve. In both cases he assumes a slip-modulus function which has been defined either for short-term loading or long-term loading.

Nailed beams

The application of the slip modulus in calculating deformations and stresses in nailed beams has been amply dealt with by Granholm, 1949 and 1961. On the basis of the work done by Granholm and Niskanen, 1961, and his own tests on nailed joints described here, the author proposes an approximated

method of calculation for uniformly nailed beams replace the method recommended in BABS 1960.

In order to illustrate the nature of the approximations introduced the application of the elasticity theory for composite beams is first summarized (cf. literature referred to above, and Larsen, 1967).

The coordinate of length is denoted x , see fig. 50. The cross section coordinate z is not influenced by the deflection y , but the z axis and the y axis coincide. In a lamina (member) are found the axial force N_i (through the centroid of the section of the lamina) and the moment M_i . The distribution of N_i over the total cross section may be expressed by equation (60):

$$N_i = \frac{dp}{dx} z_i EA_i = Ep'_i z_i A_i \tag{60}$$

z_i denotes the z coordinate of the centroid of the laminae and p is a coefficient of stress distribution in the z -direction. The axial force results in a displacement of the cross section of $\int_0^x N_i dx$. The difference in displacement between two adjoining members is thus, provided that $p = 0$ when $x = 0$:

$$\delta_i = (pz)_{i+1} - (pz)_i \tag{61}$$

At the joint there is an additional displacement caused by the bending of the members (and by the

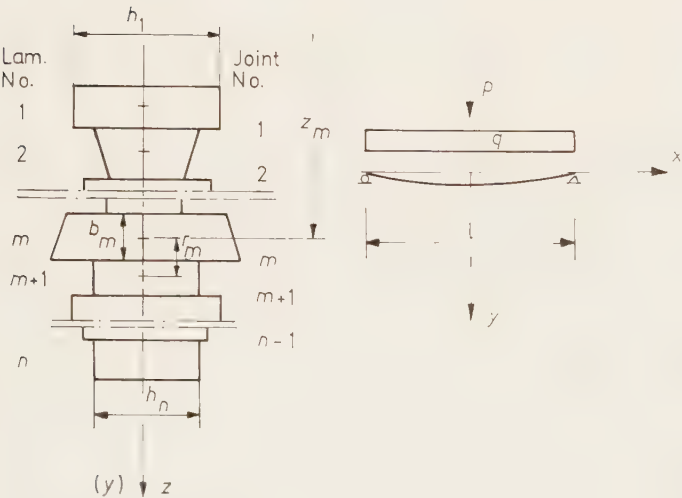


FIG. 50. Laminated beam.

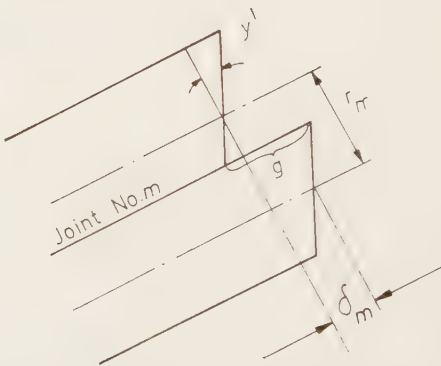


FIG. 51. Displacement of laminae.

shear of the member which is here neglected), see fig. 51. The slip in the joint then follows from eq. (62) in which $y' = dy/dx$.

$$g_i = \delta_i + r_i y' \quad (62)$$

The increase of the axial force on the length dx is offset by the shear forces of the adjacent joints, see fig. 52:

$$(T_{i-1} - T_i) dx = N'_i dx \quad (63)$$

Summing from $i = 1$ to $i = m$ with N_i according to (60) we get the force in joint no. m :

$$T_m = -E \sum_{i=1}^m p_i'' z_i A_i \quad (64)$$

Furthermore, introducing the slip modulus for the length a of the joint

$$k_m = \frac{a T_m}{g_m} \quad (65)$$

and eliminating T and g in the equations (61) to (65) we get $n-1$ equations—one for each joint:

$$-E(a/k)_m \sum_{i=1}^m (Ap'' z)_i = r_m y' + (pz)_{m+1} - (pz)_m \quad m = 1, 2, 3 \dots n-1 \quad (66)$$

In addition to (66) we have the expressions for the resulting axial force and moment in the total cross section:

$$N = E \sum_{i=1}^n (Ap' z)_i = 0 \quad (67)$$

$$M = E \sum_{i=1}^n (Ap' z^2)_i - y'' E \sum_{i=1}^n I_i \quad (68)$$

It is generally convenient to choose the level $z = 0$ through the centroid of the total cross section. If p is independent of z , the plane $z = 0$ will divide the integral $\int^A (\sigma_N + \sigma_M) dA$ into two equal parts. (This is not a neutral plane in the narrow sense of a plane where the axial stress $(\sigma_N + \sigma_M)$ is zero, except where the centroid of the total cross section coincides with the centroid of a single member). The equations may be solved after the variables p , M , y and k have been expressed in Fourier-series, see Granholm, 1961.

Approximate method

The classic assumption that a plane cross section remains plane at bending is not applicable to a composite beam with slipping members. It may be used, though, modified to the stipulation that a plane through the centroids of the members remains plane after the bending. From this stipulation (according to Niskanen, 1961, referable to Pleskov) it follows that p and derivatives of p can be extracted from the sums in (66), (67) and (68). Here the following quantities are introduced:

$$\sum_{i=1}^m (Az)_i = S_m; \quad \sum_{m=1}^{n-1} (S/r)_m = -A_r$$

$$\sum_{i=1}^n (Az^2)_i = (1 - \beta^2) I \text{ where } \beta^2 = \frac{1}{I} \sum_{i=1}^n I_i$$

Thus β^2 is the ratio of the effective moment of inertia without interaction between the members to the moment of inertia at complete interaction, i. e. to the geometric moment of inertia of the total cross section.

The denotation A_r for

$$-A_r = \frac{A_1 z_1}{r_1} + \frac{A_1 z_1 + A_2 z_2}{r_2} + \dots + \frac{A_1 z_1 + \dots + A_{n-1} z_{n-1}}{r_{n-1}} \quad (69)$$

where $z = 0$ at the centroid of the total cross section, has been used by Niskanen, 1961.

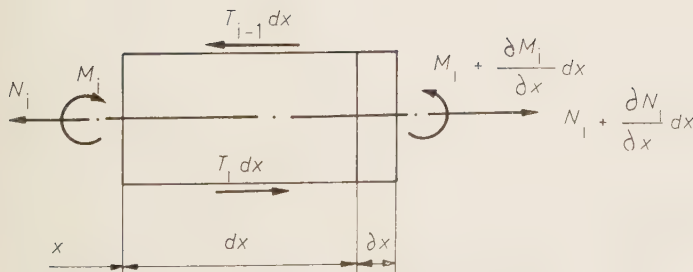


FIG. 52. Forces and moments in laminae.

As $r_m = z_{m+1} - z_m$ eq. (66) can now be written

$$\left(\frac{k}{a}\right)_m = -E \left(\frac{S}{r}\right)_m \frac{p''}{p + y'} \quad (70)$$

It is seen from (70) that p independent of z_i corresponds to a joint having $(k/a)_m = \text{const. } (S/r)_m$.

When nails are used and the spacing a between the nails is proportional to the "relative" first area moment S/r at the joint level, the method is but a special case of the above general method. In other cases, e. g. equal nailing in all the joints, the method is an approximation when there are more than two joints (symmetrical cross section).

If the equation (70) for the $(n-1)$ joints are summed, we get

$$p'' = \kappa^2 (y' + p) \quad (71)$$

where

$$\kappa^2 = \frac{1}{E} \frac{n-1}{A_r} \left(\frac{k}{a}\right) \quad (71 \text{ a})$$

In (71 a) (k/a) is the mean slip modulus of the joints. Further to eq. (71) we have from (68)

$$M = EI [(1 - \beta^2)p' - \beta^2 y''] \quad (72)$$

From eqs. (71) and (72) the differential equation for the deflection of composite beams is obtained in the form used by Pleskov among others:

$$\beta^2 EI y'''' - \kappa^2 (EI y'' + M) = -M'' \quad (73)$$

The integration coefficients belonging to (73) as usual depend on the static boundary conditions. The following solution refers to a beam on simple supports bent by a concentrated load P on the middle of the span l and by a uniformly distributed load (q per unit length) respectively. If the deflection at complete interaction between members is denoted y_0 and the axial stress (at extreme fibre) of member no. m is denoted σ_{0m} , the real deflection and stress can be written respectively

$$y = \psi y_0 \quad (74)$$

and

$$\sigma_m = \varphi \sigma_{0m} \quad (75)$$

The factor ψ , representing the magnification of the deflection, is dependent on x . In the middle of the span ($x = 0$) it is for the two loading cases

$$\psi_p = 1 + C_\psi \frac{1}{\omega^2} \left(1 - \frac{\tanh \omega}{\omega}\right) \quad (76)$$

and

$$\psi_q = 1 + \frac{4}{5} C_\psi \frac{1}{\omega^2} \left[\frac{2}{\omega^2} \left(\frac{1}{\cosh \omega} - 1 \right) + 1 \right] \quad (77)$$

where

$$C_\psi = \frac{1}{\beta^2} - 1 \quad \text{and} \quad \omega = \frac{\kappa l}{2\beta} \quad (77 \text{ a and b})$$

With the moment in a sine series, see Granholm, 1949, the magnification factors are obtained in series:

$$\psi_p = \frac{96}{\pi^4} \left(\psi_1 + \frac{1}{3^4} \psi_3 + \frac{1}{5^4} \psi_5 + \dots \right) \quad (78)$$

and

$$\psi_q = \frac{4}{5} \frac{384}{\pi^5} \left(\psi_1 - \frac{1}{3^5} \psi_3 + \frac{1}{5^5} \psi_5 - \dots \right) \quad (79)$$

where

$$\psi_j = \frac{1 - \left(\frac{\kappa l}{j\pi}\right)^2}{\beta^2 + \left(\frac{\kappa l}{j\pi}\right)^2} \quad (80)$$

Considering the difficulty of determining the correct value of the slip modulus k which is included in κ , see (71 a), it is, for practical purposes, generally sufficient to use only the first term in (78) and (79) ($j = 1$). At the same time the numerical factor before the series (78) and (79) may be approximated to unit. The deflection of the composite beam is then obtained from

$$y = \psi y_0 = \frac{1 + \nu}{1 + \beta^2 \nu} y_0 \quad (81)$$

where

$$\nu = \frac{\pi^2 E}{l^2} \frac{A_r}{n-1} \frac{a}{k} \quad (82)$$

The stress in a member with the number i is

$$\sigma_i = \frac{N_i}{A_i} + \frac{M_i}{EI_i} (z - z_i) \quad (83)$$

Assuming that p is independent of z_i , M_i is expressed by (72) and N_i by (61). Introducing the factor

$$C_i = \frac{1}{\beta^2} \left(1 - \beta^2 - \frac{z_i}{z} \right) \quad (84)$$

we can then write eq. (83)

$$\sigma_i = \varphi_i \sigma_{oi} = \left[1 + C_i \frac{\beta^2}{1 - \beta^2} \left(\frac{y''}{y_0''} - 1 \right) \right] \sigma_{oi} \quad (85)$$

In (85) $\sigma_{oi} = M/Iz$ is the stress and $y_0'' = M/EI$ is the curvature at complete interaction between the members. The curvature of a beam with slipping members y'' is derived from the expression for y_x — not given here—from which the deflection in the middle $y_{x=0}$ has been calculated according to equa-

tions (76) and (77). The corresponding magnification factor of the axial stress is for concentrated load and uniform load respectively

$$\varphi_{ip} = 1 + C_i \frac{\tanh \omega}{\omega} \quad (86)$$

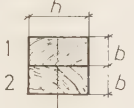
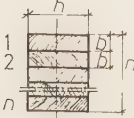
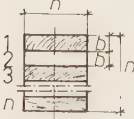
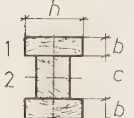
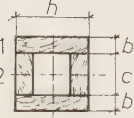
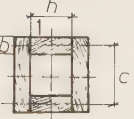
$$\varphi_{iq} = 1 + C_i \frac{2}{\omega^2} \left[1 - \frac{1}{\cosh \omega} \right] \quad (87)$$

The factor ω is defined by

$$\omega^2 = \frac{l^2}{4\beta^2 E} \left(\frac{k}{\alpha} \right) \left(\frac{n-1}{A_r} \right) \quad (88)$$

in which β^2 is the ratio of the sum of the individual moments of inertia to the moment of inertia of the total cross section and k/a the slip modulus. Table 19 shows the value of the cross section factor $\frac{A_r}{n-1}$ for beams of a common type. The coefficient C_i is dependent on the cross section coordinate z ($z = 0$ in the centroid of the total cross section).

TABLE 19. Quantities for calculating deflection and bending stresses in nailed beams.

Cross section	$n-1$	$\frac{A_r}{n-1}$	C_1	C_2
	1	$\frac{bh}{2}$	1	1
	$n-1$	$n(n+1) \frac{bh}{12}$	$C_i = \frac{n^2}{n-2(i-1)} - 1$	
	$n-1$	$(n+1)(n+3) \frac{bh}{24}$	$C_i = \frac{1 + \frac{48S}{n+1}}{n-2(i-1)} - 1$	
	2	bh	$\frac{1}{\beta^2} \frac{b}{2b+c} - 1$	$\frac{1}{\beta^2} - 1$
				
				

S from eqs. (90) or (91).

The C -values found in table 19 refer to stresses at extreme fibre of the member in question. Thus, $\mp 2(z-z_i)$ gives the thickness of the member.

When the members are separated, as shown in the third case in the table.

C_i is obtained from

$$C_i = \frac{1 + \frac{48 S}{n + 1}}{n - 2(i - 1)} - 1 \tag{89}$$

In equation (89) S stands for the sum according to (90) or (91):

If $\frac{n+1}{2}$ is an even number, S is

$$S = 1^2 + 3^2 + \dots + \left[\frac{n-1}{2}\right]^2 \tag{90}$$

and if $\frac{n+1}{2}$ is an odd number, S is

$$S = 2^2 + 4^2 + \dots + \left[\frac{n-1}{2}\right]^2 \tag{91}$$

The values of k proposed in table 20 are directly related to the values according to table 18. However, the value of k decreases at high stress. This is usually taken into account by the requirement that the distance between the nails should not ex-

TABLE 20. Slip modulus (k) for calculating ω^2 according to eq. (88). Nail diameter d cm.

Type of load	k kp/cm	Maximum permissible value of k
A. Long-duration load (dead load, snow load, etc.)	1200 d	700
B. Short-duration load (live load, load on temporary structures, etc.)	2000 d	1200
C. Wind load	2700 d	1600

ceed a certain limit. According to BABS 1960 (and earlier editions) and also according to DIN 1052, nailed joints shall be dimensioned on the basis of the shear stresses that would occur in the joints at complete interaction between the members. Here this requirement has been slightly modified by permitting the working load to be exceeded by 40 % (exceptional load). Thus, the values of k in table 20 should preferably not be applied if the distance a exceeds

$$a_{\max} = \frac{1.4 P_w}{R} \cdot \frac{I}{S_z} \tag{92}$$

In (92) P_w is the (permissible) working load per nail, $R = dM/dx$ the transverse shear force and S_z and I the first and the second area moments. It must be observed that a denotes the distance between fully active nails. If there is more than one row of nails $a = 1/N$, where N is the total number of

TABLE 21. Relative moment of inertia (for calculating deflection) and maximum stresses in flange and web in a nailed beam, fig. 53. Grooved wire nail 43/125. Span 4 m, concentrated load at the centre of the span.

Referents	Distance between nails	Load ^a P	Force per nail ^b S	Slip modulus k	Relative moment of inertia I_e/I	Maximal stresses	
	a					Flange	Web
	cm	kp	kp	kp/cm	%	kp/cm ²	kp/cm ²
A. Test results according to Granholm, 1949	4	600K	51	1900	52	55	93
	7.5	600K	80	1400	46	55	96
B. BABS 1950, 1960	4.6	780L	70	325	30	(100) ^c	(70) ^c
C. Author's proposal	4.6	860K	77	2700 d =1160	52	60	140
	7.5	660K	98	1160	42	60	112
	4.6	480L	43	1200 d =520	37	45	101
	7.5	410L	60	520	30	45	101
D. DIN 1052/1964	4.6	640L	57	600	48	59	100
	7.5	480L	70	600	32	44	94

^a In the cases B to D, P is the value of the maximum working load—short-duration (K) and long-duration (L)—according to the stipulations of the code. It is dependent either on the maximum working load per nail or the maximum working stress in the timber (values in italics).

^b In the cases B to D the shear force is calculated according to BABS and DIN without regard to the slip between the members.

^c These stresses—which according to BABS 1960 have been assumed to be 0.7 times the stresses at complete interaction—are quite erroneous. At the relative effective moment of inertia assumed, also according to the norms, ($I_e/I = 0.30$), the bending stress in the web would be 190 kp/cm² (1900 N/mm²), i.e. about twice as high as the working stress allowed at long-term loading.

active nails on the length l of the joint. It should also be observed that the distance between the nails ($a_{\max}$) in (92) is not explicit, since either R or I/S is a function of a , depending on whether the calculation is based on specified timber dimensions or—which is generally the case—on a specified load (R).

Example:

In the following example, the timber dimensions are borrowed from a beam (fig. 53) tested by Granholm, 1949. According to table 21 even a comparatively great distance between the nails, Granholm's tests (A), resulted in a somewhat higher degree of interaction between the members (I_e/I) than the interaction corresponding to the k value, 2700 d , proposed as short-term value in table 20. The load per nail at these tests did not exceed the maximum value stipulated in BABS.

Calculated according to BABS 1960 (B in table 21) a distance between the nails of $a = 4.6$ cm gives full utilization of the nails as well as the timber. The more realistic method proposed by the author (C in table 21) gives lower values of the force on the nails and the stresses in the flanges, but higher stress in the web, as mentioned in the footnote, to the table. The long-duration load, 780 kp (7800 N), according to BABS 1960, is consequently too high.

According to the DIN proposal (D in table 21) a load of 640 kp (6400 N) may be permitted without causing the stress in the web exceed the permissible value, 100 kp/cm² (10 N/mm²) for the timber in

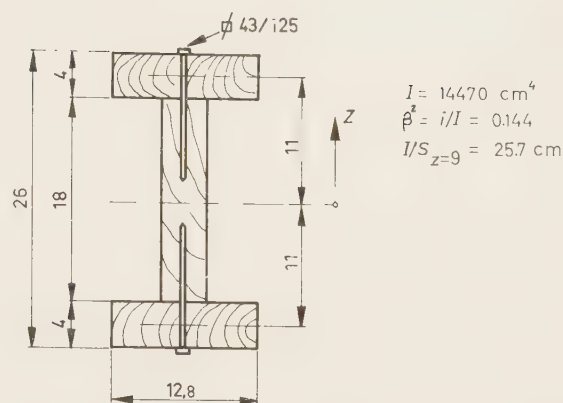


FIG. 53. Cross section of nailed beam tested by Granholm (1949) and used by the author in applying different methods of calculation, table 14. (Measures in cm.)

question. Also, the magnification factor for the stress in the web is assumed to be equal to I/I_e , i. e. equal to the magnification factor for the deflection at the centre. In reality, it is slightly higher in the case of concentrated load at the centre (cf. Granholm, 1949) and the actual stress in the web at a load of 640 kp (6400 N) is 127 kp/cm² (12.7 N/mm²). Consequently, the concentrated load should not have been permitted to exceed 500 kp (5000 N). According to proposal C, the permissible long-duration load at a distance between the nails of 4.6 cm is 480 kp (4800 N), thus even slightly lower, because the displacement modulus at the nail diameter concerned is somewhat lower than the value according to DIN 1052. Had the load been evenly distributed, the proposal C would instead have resulted in a slightly (3 %) higher working load than the DIN 1052 proposal.

SAMMANFATTNING

Spikförbandens hållfasthet och eftergivlighet är en komplicerad funktion av träets och spikarnas egenskaper. Proportionalitet mellan belastning och förskjutning existerar strängt taget inte. Förskjutningen är vidare starkt beroende av belastningstiden. Av detta följer att en noggrann beskrivning av spikförbandens verkningssätt, då de belastas under olika villkor, med nödvändighet kommer att avvika från de generaliserade och enklare uttryck för påkänningar och deformationer som kan komma ifråga i rekommendationer och normer för konstruktörernas arbete. Den grundläggande förbandsforskningen har till ändamål att bestämma gränserna för de approximerade uttryckens giltighet och att finna modifikationer för särskilda användningsvillkor. Som regel är första målet för sådan forskning att fastställa sambandet mellan förbandens dimensioner, därvid speciellt förhållandet mellan virkestjocklek och spikdiameter, samt förbandets mekaniska egenskaper. När det gäller små rörelser kan detta förhållande någorlunda tillfredsställande anges inom ramen för elasticitetsteorin och för stora rörelser med plasticitetsteorin. Problemet blir betydligt svårare för mellanliggande deformationer.

Vare sig kraft/förskjutningskurvan är delad i tre delar, åtskilda av proportionalitetsgräns och flytgräns, eller betraktas som en kontinuerlig funktion, är den definierad av parametrar som vid givna virkes- och spikdimensioner är jämförbara med de grundläggande materialkoefficienterna för elasticitet, viskositet och styrka. Parametrarna är emellertid av teknologisk art och svåra att ställa i relation till elasticitetsmoduler, hållfasthetsvärde etc. enligt standardprovning. Detta framgår bl. a. av förhållandet att träets styrka under belastning från spikar är beroende av spikarnas diameter och tvärsnittsform.

De i rapporten beskrivna experimenten avser i första hand de av träegenskaperna beroende parametrarna och motsvarar direkt förhållandena i förband där spikarnas krökning kan negligeras. I rapportens första avsnitt företes resultat av korttidsbelastning. Resultatet anges med karakteristiska

spänningsvärden (bäddmodulen K , kp/cm^2 , definieras som den spänning vilken efter rätlinjig extrapolering av kraft/förskjutningskurvans första del motsvarar en intryckning av spiken i träet lika med spikdiametern). Inflytandet av spikens diameter och tvärsnittsform, träets täthet och fuktkvot vid spikning och provning samt inflytandet av kraftriktningen på kraft/förskjutningskurvans karakteristiska spänningsvärden kontrollerades med variansanalys. Inflytandet av virkestätheten befanns därvid vara oberoende av andra variabler och lika med täthetens inflytande på träets tryckhållfasthet i fiberriktningen vid provning på standardprismor. Som omräkningsfaktor föreslås ($3r_{0u} - 0,33$) där r_{0u} betecknar träets täthet, g/cm^3 , i torrt tillstånd. De andra variablerna samverkar på olika sätt. Till exempel kunde inget generellt gällande inflytande av spikdiameterns storlek på bäddmodulen fastställas. Däremot visade kompletterande prov med bultar i förborrade hål (diameter 8—25 mm) en med diametern nära proportionell bäddmodul. Vid högre spänningsnivåer var spikdiameterns inflytande betydande, speciellt vinkelrätt mot fibrerna. Vid 5 mm spikdiameter var trähållfastheten ungefär densamma parallellt med som vinkelrätt mot fibrerna. Eftersom diameterinflytandet var negativt och störst vinkelrätt mot fibrerna, skall tillåtna spänningar, som grundas på kraft/förskjutningskurvans slutdel och som är avsedda för användning oberoende av vinkeln mellan kraft och fibrer, grundas på resultaten erhållna vid tryck i fiberriktningen. [Andra provningar har visat att hållfastheten i riktning 45° mot fibrerna inte understiger styrkan i fiberriktningen (Siimes *et al.*, 1954).] Däremot skall tillåtna krafter för bultar grundas på resultat av provningar vinkelrätt mot fibrerna, såvida inte särskilda modifikationsfaktorer för vinkeln mellan kraft och fibrer införes. I BABS 1967 har för bultförband introducerats ett diameterinflytande som beror på vinkeln mellan kraftriktning och fiberriktning.

Inflytandet av fuktkvoten på bäddmodul och övriga bestämda spänningar var mera komplicerad än

inflytandet av tätheten. I medeltal erhöles 40—50 % lägre värde då förbanden provades i vått tillstånd jämfört med då de provades vid 12 % fuktkvot. Resultaten från prov, som spikats i vått tillstånd men provats efter konditionering till 12 % fuktkvot (F_3), resulterade i spänningvärden motsvarande dem som erhöles, då provet spikades och provades i torrt tillstånd. Undantag utgjorde bäddmodulen, som inte ökade signifikant efter torkning. Runda spikar gav lägre bäddmodul och hållfasthet än räfflad trådspik. Hållfastheten i fiberriktningen under spikar med liten diameter närmade sig träets tryckhållfasthet i fiberriktningen enligt provning av standardprismor.

I nästa del av rapporten återges resultat av långtidsbelastning av spikförband av i princip samma typ som de korttidsbelastade. För den fenomenologiska analysen av resultaten användes den välkända reologiska modellen av Kelvin-element kopplade i serie. Rörelseekvationen generaliserades dock med en korrektionsterm som innehöll parametern α för bestående deformation och β för avvikelser från linjäritet. Resultaten av dessa prov, vid vilka förbanden belastades direkt till vilonivå, visade att den "omedelbara" deformationen inte är proportionell mot belastningen. (Olinjäritet i ett spikförbands kraft/förskjutningskurva vid vanlig korttidsprovning behöver inte betyda att reologisk olinjäritet föreligger.) Krypningen vid första pålastning var betydligt större än proportionell mot spänningen, även större än proportionell mot den omedelbara deformationen. Förhållandet mellan krypning och omedelbar deformation var således inte konstant såsom i ett linjärt material. Den relativa krypningen i dessa förband i vilka spikarna parallellförflyttades i träet kan enligt resultaten bäst beskrivas som proportionell mot den omedelbara intryckningen. Den omedelbara intryckningen av spikar med 3,2 mm diameter var i medeltal 0,08 mm vid en hålkantspänning av 100 kp/cm² (10 N/mm²) och krypningen uppgick till 40 % efter två månaders vilande belastning. Vinkelrätt mot fibrerna uppgick den omedelbara intryckningen under samma hålkantspänning — som är ganska låg för praktiska förhållanden — till 0,16 mm och krypningen efter 2 månader till 80 % av detta värde, dvs. krypningen vinkelrätt mot fibrerna var 4 ggr större än i fiberriktningen. Deformation av kraft i fiberrikt-

ningen återhämtades oberoende av krypningens storlek till mycket liten del efter avlastning. Enligt resultaten bör återhämtningen inte räknas bli större än som motsvarar bäddmodulen. I proven belastade vinkelrätt mot fibrerna återgick korttidsdeformationen nästan fullständigt, under det att ungefär 1/3 av långtidskrypningen var elastisk. Deformationerna av intermitterent belastning överensstämmer approximativt med den enkla modell, som författaren använt för att dela deformationen i reversibla och irreversibla komponenter.

Rapportens tredje avsnitt ägnas förskjutningen i spikförband av den typ som i allmänhet förekommer i praktiken, dvs. förband i vilka, i motsats till vid de i det föregående beskrivna proven, hålkanttrycket varierar längs spiken. Den omedelbara rörelsen och krypningen i sådana förband jämförs med resultaten av intryckningsprovningarna. I ett tidigare arbete har författaren visat, att krypningen i spikförband teoretiskt borde retarderas genom spikens böjning. Detta bekräftas av försöksresultaten. Trots det faktum att hålkanttrycket under spikarna i några av de av författaren provade förbanden nådde 400 kp/cm² (40 N/mm²), motsvarande en intryckning av spiken i träet av 1,4 mm, stannade den uppmätta krypningen i förbanden vid ett värde som ungefär motsvarar krypningen i proven med konstant hålkantspänning, men en initialintryckning av endast 0,2 mm. Spikarnas böjning gav vidare förbanden en elasticitet i krypningen av omkring 40 % till vilken ingen motsvarighet erhöles vid intryckningsprovningen.

Vid statiska beräkningar för dimensionering av träkonstruktioner tages hänsyn till deformationerna med införande av elasticitets- och skjuvmoduler för träet. På motsvarande sätt tages hänsyn till förskjutningar mellan virkesdelarna genom införande av förskjutningsmoduler. I rapportens slutavsnitt anges förskjutningsmoduler för spikade förband, grundade på försöksresultaten. Verkliga och skenbara värden på förskjutningsmodulen diskuteras och modifierade värden för användning inom elasticitetsteorin föreslås. Som användningsexempel visas en förenklad metod för beräkning av spänningar och deformationer i sammansatta spikade balkar, grundad på arbeten av bl. a. Granholm och Niskanen.

SUMMARY

The strength of nailed joints and the deformation and slip in such joints are complicated functions of the properties of the wood and the nails. The load slip relation deviates from the straight line at very a low load level and the shape of the curve depends on the loading time. For this reason a true description of the behaviour of nailed joints, loaded under different conditions is bound to diverge from simple values or formulas for design loads and slip in the joints given in building recommendations and codes. However, fundamental research is necessary if we are to be able to determine the limits of applicability of the approximated formulas used in practice and to modify them to suit special conditions. As a rule, the first aim of such studies is to establish the relationship between the dimensions of the joint, particularly the ratio of timber thickness to nail diameter, and the mechanical properties of the joint. This is comparatively simple as regards small deformations (elasticity theory) and flow in the joint (plasticity theory). It is considerably more difficult as regards the interjacent part of the load-slip curve.

Whether the load-slip curve is divided into three parts, separated by the limits of proportionality and flow, or a continuous function, it is defined by parameters which at given timber and nail dimensions are comparable to elementary material coefficients (elasticity and viscosity coefficients and strength). These parameters, however, are of a technological nature and cannot easily be derived from elasticity moduli, strength values, etc. determined by standard tests. This is clear from, among other things, the fact that the strength of wood when loaded from nails is dependent on the diameter and section of the nails. By means of lateral displacement tests the author has determined the coefficients which primarily depend on the properties of the wood. The results of short-duration loading are described in the first part of the report (p. 9). The coefficients are expressed as stresses. (The displacement modulus K , may be defined as the stress

which—the first part of the load slip curve having been extended linearly—corresponds to a displacement equal to the diameter of the nail). The influence of the diameter and section of the nail, the density and the moisture content and the direction of the load on defined stress values along the load-slip curve was checked by means of variance analysis. The influence of the density was found to be independent of the other variables, and equal to the influence of the density on the compressive strength of wood parallel to the grain according to tests on matched prisms. As conversion factor for the displacement modulus as well as the stress values along the load-slip curve ($3 r_{0u} - 0.33$) is suggested where r_{0u} denotes the density of absolutely dry wood in g/cm^3 . As regards the other variables there are several interactions. No general influence of the nail diameter on the displacement modulus could therefore be determined. On the other hand, supplementary tests with bolts in pre-bored holes (diameters 8 to 25 mm) showed a displacement modulus almost proportional to the diameter. The diameter dependence at high stress levels, e. g. the stress that gives 2 mm displacement, was considerable perpendicular to the fibres and also significant parallel to the fibres. At large nail diameters (5 mm) the compressive strength was of the same order parallel as perpendicular to the grain. As the diameter influence is negative and greater perpendicular to the fibres, design stresses based on the final part of the load-deformation curve and intended for use independent of the angle between the force and the fibres should be based on the result obtained parallel to the fibres. [According to other tests (Siimes *et al.*, 1954) the strength at an angle of 45° between force and fibres is not lower than the strength parallel to the fibres.] On the other hand, design loads for bolts should be based on tests perpendicular to the fibres, provided that the loads are not modified with respect to the direction of the force. In the Swedish Building Code (BABS 1967) a diameter influence is introduced which is dependent on the angle between the force and the fibres.

The influence of the moisture content on the displacement modulus and the other stress values is more complicated than the influence of the density. On an average 40 to 50 % lower values were obtained when the wood was soaked at testing than when it had a moisture content of 12 %. The results from specimens nailed in a soaked condition but tested after drying to 12 % moisture content (F_3) gave stress values similar to those obtained with specimens nailed and tested in a dry condition. However, the displacement modulus did not increase significantly at drying.

Round nails gave lower displacement modulus and strength than grooved nails of the type used in Sweden.^a The strength parallel to the fibres under nails with small diameters approached the compressive strength determined on matched control prisms of standard size.

The second part (p. 34) deals with results from creep tests on joints corresponding to those described in the first part of the report. At the phenomenological treatment of the results the well-known rheologic model of Kelvin bodies in series was used. However, the equation for the displacement was generalized by terms which include the parameters α for irreversibility and β for the deviation from linearity. Non-linearity of the load-slip curve of a nailed joint does not preclude rheologic linearity. However, the creep tests showed that non-linearity appears already in the instantaneous displacement. Furthermore, the displacement creep increases considerably more than proportionally to the stress and also more than proportionally to the instantaneous displacement. Thus, the ratio of creep to instantaneous deformation is not constant as in a linear body. This relative creep, according to the results should rather be described as proportional to the instantaneous displacement. The instantaneous displacement of 3.2 mm nails was about 0.08 mm and the creep 40 % after two months of 100 kp/cm² (10 N/mm²) compressive stress. This is a rather low stress for a nailed joint (in practice). Perpendicular to the fibres the same instantaneous displacement was 0.16 mm and the creep after two months 80 %, i. e. four times the creep in the fibre direction. The

recovery of the displacement in the fibre direction, after the load was removed, was very small, independent of the size of the creep. In practice only the displacement corresponding to the displacement modulus should be expected to recover. Perpendicular to the fibres the short-term displacement recovered almost completely, while about one third of the long-term creep was elastic. The result of intermittent loading was in agreement with the simple model which the author used to divide the displacement in reversible and irreversible components.

The third part (p. 51) is devoted to slip in nailed joints of the type used in practice, i. e. joints where, contrary to the conditions at the test previously described, the compressive stress varies along the nail. The instantaneous slip and creep found in such joints are compared with the results of the displacement tests. In a previous paper the author showed that the creep in the joint should, theoretically, be retarded by the bending of the nail (Norrén, 1962). This is confirmed by the test results. Despite the fact that the stress under the nails in some of the joints tested by the author reached 400 kp/cm² (40 N/mm²), equivalent to an instantaneous displacement of 1.4 mm, the measured relative creep of the slip was of about the same order as the creep at the displacement tests after an instantaneous displacement of only 0.2 mm. The bending of the nails also gave the joints an elasticity of creep of about 40 %, which was not found at the displacement tests.

When timber structures are designed, deformations in the wood are generally taken into account by the introduction of elasticity moduli and shear moduli. In the same way shear between the timber members is taken into account by slip moduli. In the fourth section (p. 61) the author gives slip moduli for nailed joints based on the displacement moduli determined. True and apparent values of the slip modulus are discussed and values to be applied in formulas derived from the elasticity theory, but modified for creep, are suggested. As an example of application the author shows a method based on studies by, among others, Granholm and Niskanen, by which stresses in nailed composite beams and the deflection of such beams can easily be calculated.

^a Longitudinally grooved = fluted.

New literature on nailed joints

After delivery of the manuscript, several reports and papers on nailed joints have been published. It is not possible to give a full account here, but a few reports within the scope of the author's tests have been selected and commented upon.

Displacement and bearing capacity of nails at short-term loading

The properties of nailed joints at short-term loading have been investigated by several authors. The performance of round and Swedish grooved nails has been studied in single-shear joints of Douglas Fir (Sholten & Doyle, 1966). The loads giving 0.4 and 2.5 mm slip proved 20 to 30 per cent higher in joints with grooved nails than in joints with round nails of equal cross-sectional area. The slenderness ratio (timber thickness to nail diameter) being comparatively high, this difference in stiffness may be estimated by the use of eqs. (54a) and (54b). If, in accordance with fig. 6, the displacement modulus for grooved nails is assumed to be 25 per cent higher than for round nails, the slip modulus should also be about 25 per cent higher than for the round nails. Correspondingly, a difference in strength of 30 per cent can be calculated by applying current theories (Johansen, 1941, Möller, 1951, and others), if the timber strength under the round nails—see fig. 11—is assumed to be 2/3 of the strength under the square grooved nails. Of these 30 per cent, 10 per cent relate to the different bending stiffness of the two types of nails. The test results further indicate a slight diameter dependence of the different moduli of stiffness and strength but, in view of the rather limited number of tests, a quantitative comparison with the results given in the first section, pp. 9—33, is not justifiable.

In similar tests (Kay, Hall & March, 1965) plain nails with round, square (not grooved) and triangular cross sections of approximately similar area have been compared in joints of European redwood (*Pinus Silvestris*). The ultimate loads—probably ob-

tained at large slip—did not deviate much, though the joints with round nails proved significantly weaker than the joints with square or triangular nails. The slip modulus, calculated as secant modulus from 0 to 0.4 mm slip, was 40 per cent lower in joints with round nails than in joints with square nails. Expressed in nail diameter (d in cm), the corresponding slip moduli were $3000 d$ for square nails, and $1500 d$ for round nails (cf. table 18). Though the square nails tested are supposed to provide joints of higher strength (flow load) than the grooved nails with the same diameter (Johansen, 1965), the difference between the slip moduli is greater than might have been foreseen from the results given in first section. The placing of the square nails showed no effect on the slip or the strength, whereas nails with a triangular cross-section had the lowest slip modulus but the highest ultimate load placed with one side parallel to the force and the fibres.

The influence of knots on the bearing capacity of nails has been investigated by Egner & Kolb, 1963. Unfortunately, too many nails were used in the joints. Many of the joints failed in the timber before the bearing capacity of the nails was reached. Moreover, few nails were driven in knots. Thus, hardly any significant conclusions can be drawn about the influence of knots. The load-slip curves presented in the same paper, which refer to round nails in pine and spruce at 15 per cent moisture content, show that the slip (secant) modulus at half working load may be estimated to be $5000 d$ and $5900 d$ at a nail diameter of 3.4 and 4.2 mm respectively. The corresponding moduli based on working load are $1600 d$ and $2000 d$. The joints were repeatedly loaded and unloaded at working load level. The slip at the repeated loading cycles can be approximately expressed by the modulus for the slip at first loading to half the working load.

Another paper (Möhler, 1963) gives load-slip curves and ultimate stress values for joints of beech plywood nailed to timber (species not specified). Ac-

cording to the curves, the slip modulus at working load varies between $1700 d$ and $4400 d$ (values obtained at 11 per cent moisture content). The ratio between plywood thickness and nail diameter in the tested joints was 3.2 or higher. The ultimate load obtained was at least three times the working load stipulated in the German code (DIN 1052) for timber members with a thickness of at least $8 d$. Thus, the report concludes that these working loads are applicable also to the plywood joints tested, though this might be questioned, considering the large deformation at ultimate failure. In many countries, including the Scandinavian, the working loads in timber joints are based on the load at flow (yield).

The influence of the plywood thickness on this load may be demonstrated as follows:

In an ordinary three-member joint with a main member sufficiently thick to cause a sharp nail bend, there is a critical side member thickness l_R below which the nail will not bend in the side members. At l_R the load giving "flow slip" in the joint begins to decrease with decreasing timber thickness. The value l_R coincides approximately with that given in codes of practice ($7 d$ to $8 d$), below which thickness the working load of the nails is supposed to be reduced. The corresponding flow load is $\sqrt{2qM}$ per nail and joint. Let us now assume that the side members of wood are replaced by members of plywood having twice the timber's strength, i. e. the embedding strength $2q$. (This is in conformity with the strength ratio between the beech plywood and the timber used by Möhler). Formulas for the flow load (cf. Norén, 1962) show that, if plywood of the thickness $0.5 l_R$ is chosen, the strength of the joint remains unchanged. However, theoretically, this plywood thickness is not sufficient to cause nail bending in the side members. This will not occur below a plywood thickness of $0.65 l_R$, at which the bearing capacity of the nail in the plywood/timber joint reaches its maximum value $(2/\sqrt{3}) \sqrt{2Mq}$.

Figure 54 shows results of nail displacement tests in Swedish pine plywood, carried out as described in first section. For the different levels of nail diameter the plywood thicknesses 8.1, 12.6 and 20.4 mm were used. The fact that the load could be increased considerably above the value giving 2 mm displacement—contrary to the case of timber (parallel to the fibres)—seems to justify the assumption that plywood has twice the embedding strength of

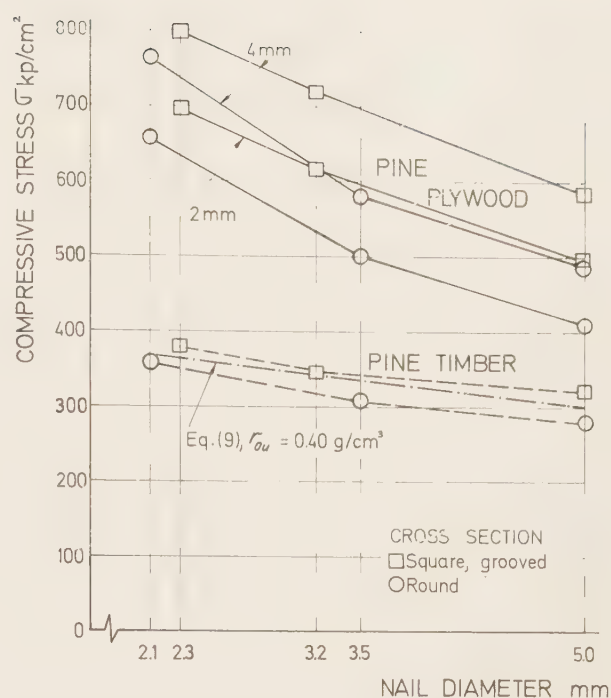


FIG. 54. Stress giving 2 and 4 mm displacement in Swedish pine plywood ($r_{0u} = 0.50 \text{ g/cm}^3$, $u = 12 \%$). Mean values from 18 specimens for each type of nail and diameter. Half the number of specimens were tested with the force parallel and half the number with the force perpendicular to the fibre direction of the face veneers, but as the difference between the mean values was less than 10 per cent, the results have been pooled. The lower curves show the stresses (parallel to the fibres) which according to first section give 2 mm slip in pine.

pine timber. This would mean that the timbers used for splicing may be replaced by plywood of half the timber thickness, i. e. $3.5 d$ according to the Swedish building regulations (1967). In fact, a slightly lower value has been suggested by the recommendation that the nail diameter should not exceed $1/3$ of the plywood thickness (Jansson, 1965).

The grooved nails were also tested with the cross-section parallel to the force. Hereby approx. 10 per cent improvement of the strength values presented in fig. 54 was obtained. Furthermore, the displacement modulus was determined. The mean result was 2300 kp/cm^2 (230 N/mm^2) for nails with round and 3100 kp/cm^2 (310 N/mm^2) for nails with square (grooved) cross-section. These values are lower than those given for timber in first section. A value $K = 3000 \text{ kp/cm}^2$ (300 N/mm^2) according to eq. (54a), applicable to nails with round or plain square cross-section, will give a slip modulus $k = 2500 d$ in a plywood-to-ply-

wood joint, provided that the plywood thickness is at least $l = 3.4 d$. The corresponding k -value for $K = 5000 \text{ kp/cm}^2$ (500 N/mm^2) is $3700 d$ at $l \geq 3 d$.

Plywood-to-plywood joints have in fact been tested (Kangas, 1966). Finnish birch-plywood was used in three-member joints. From the load-slip curves, which are in most cases fairly straight to a rather high load level, slip moduli varying from approx. $2200 d$ to $5200 d$ can be derived, the lowest values referring to joints tested in tension.

Nail displacement tests have also been carried out as part of a comprehensive Norwegian study of nailed joints (Aune, 1966). At these tests, nails with square cross-section in pine and spruce were loaded parallel to the fibres, mainly to determine the ultimate embedding strength. Though the rate of displacement is not specified, and the displacement at ultimate load varies between 3 and 7 mm, the embedding strength values may be compared with the strength values given in first section. Nails with square cross-section, with and without longitudinal grooves, were used, the former of a type similar to the nails denoted S_2 in first section; the latter ones with blunt edges may be placed somewhere between S_1 and S_2 with regard to type.

Thus, the result that the strength was highest under grooved nails placed diagonally (S_3), but the influence of the cross-section shape otherwise insignificant, confirms the result illustrated in fig. 11. The ultimate embedding stress was slightly higher when the nails were driven radially to the annual rings than when they were driven tangentially or diagonally. This is in conformity with the results referred to on p. 10 (Siimes *et al.* 1954). An average ratio of ultimate strength under nails to compressive strength of control prisms of approx. 0.85 was obtained (cf. fig. 18).

Yield theory

The ultimate lateral force that a slender nail, at ideal plastic conditions transfers from one timber member to another is

$$P = 2q_f M_f \quad (93)$$

The expression is based on the assumption of constant stress (q_f , kp/cm) from the joint to the point where the bending moment in the nail reaches its ultimate value. If, however, the suffix f —instead

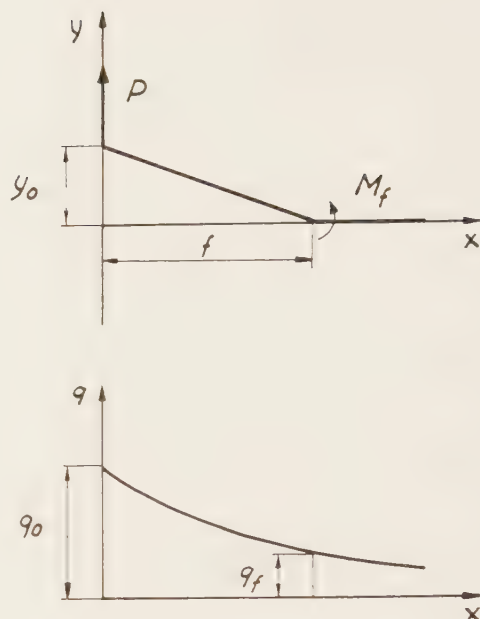


FIG. 55. Bending of nail at yield.

of indicating flow value—merely indicates the distance from the joint ($x = f$, see fig. 55), the stress in the timber may be assumed to increase above q_f . It may be expressed in terms of the displacement (y), for example in accordance with

$$q = q_f y y_f^c$$

If the moment is assumed to have a maximum value M_f (not necessarily a flow value) at $x = f$, the equilibrium equations are obtained from the fact that at that point the shear force in the nail is zero.

$$S = \int_0^f q_f \left(\frac{f-x}{f} y_0 y_f \right)^c dx \quad (94)$$

$$M_f = fS - \int_0^f q_f \left(\frac{f-x}{f} y_0 y_f \right)^c (f-x) dx \quad (95)$$

If f is eliminated from (94) and (95), a relation between S , y_0 and y_f is obtained. If, furthermore, $\log q$ and $\log y$ are assumed to be linearly correlated and the ratio $q_f y_f^c$ is replaced by a displacement modulus K , the force S is obtained expressed in y_0 (which is half the slip in the joint):

$$S = \left[\frac{c+2}{c+1} K y_0^c M_f \right] \quad (96)$$

At $c = 1$ and $K y_0 = q_f$ eq. (96) is transformed into (93).

Expressions similar to (96) have been derived by Aune for the actual symmetrical case and for other cases of nail deformation (Aune, 1966). Instead of using the equilibrium equations (94) and (95), Aune applied an energy method including the condition $dS/dx = 0$. However, by treating f as a constant in integrating the energy generated by a virtual displacement, Aune obtains a c -dependent coefficient deviating from the coefficient based on (94) and (95), except when $c = 1$.

Aune also presents further generalizations by introducing a maximum moment at $x = f$ which is proportional to the deflection angle of the nail at this point. This renders it theoretically possible to use the derived expressions even at small loads on the joint. Still, the expressions are not likely to describe the complete load-slip curve. The best value for describing the shape of the load-slip curves of the tested three-member joints was $c = 1/4$. The second parameter, here introduced as a displacement modulus K , is by Aune substituted by the ratio $q_{\max}/y_{\max}^c$, referring to the ultimate stress and displacement obtained from the displacement tests. The calculated load-slip curves of the nailed joints, consequently, best agreed with the test results at relatively high loads. At low loads, the agreement was less good, which, apart from the choice of parameters, may to some degree depend on y not being proportional to x along the distance f .

Creep in nailed joints

In part III of the report "Holzbau-Versuche", Möhler presents results from long-term loading of round nails (diam. 3.1, 4.2 and 6.0 mm) in three-member joints of spruce (Möhler, 1966). From the slip at short-term loading, a slip modulus varying from $2400 d$ to $15,000 d$ is obtained, values between $3000 d$ and $5000 d$ being in majority. The highest values are probably influenced by friction, since storing of joints over several months before testing made the slip moduli decrease due to shrinkage to normal (and subnormal) values. The corresponding displacement moduli, calculated by means of eq. (54a) vary from 2800 to 32,000, mainly between 3000 and 6000 kp/cm^2 (600 N/mm^2). In some series the nails were driven into prebored holes whereby, in most cases, some increase in the moduli was obtained. Of special interest is the creep obtained under long-term load. The level of constant loading varied from working load to 1.5 times the working load between series of joints. As seen from

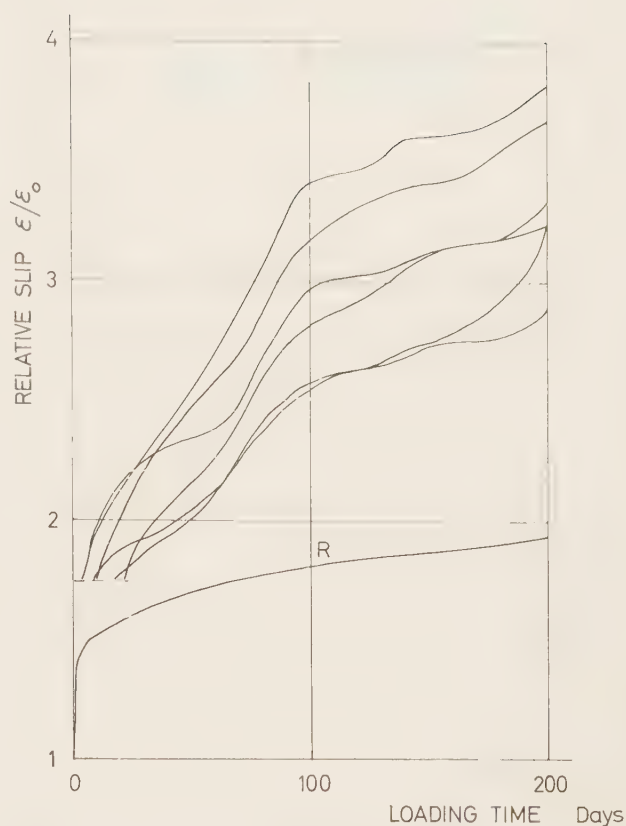


FIG. 56. Creep in nailed joints, loaded 40 to 50 per cent over working load. Decreasing moisture content during test. (Möhler, 1966). R is the reference curve defined by eq. 35.

fig. 56, creep was at least twice the creep obtained by the author (cf. figs. 47 and 48). There is little doubt that the reason for this is variations in the relative humidity and, particularly, the reported decrease in moisture content during loading. Anyway, in one series of six joints, in which the moisture content is supposed to have stabilized after some days of loading, the creep rate after a few weeks was very small. The total slip after 200 days was about 2.5 mm, while after the same time the total slip was 5 to 9 mm in joints where the moisture content was allowed to decrease from about 20 to 15 per cent during loading. Of course, the variations in moisture content, though of practical importance, make it very difficult to analyse the influence of other variables, such as nail diameter, preboring, stress level, etc. After removal of load and recovery, the long-term tested joints were loaded to failure in a short time. As could be expected, the ultimate loads thus obtained did not deviate significantly from the ultimate load obtained at short-term loading of matched joints without a loading history.

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